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Experimental Investigation of Sloshing Wave Effects on a Fixed Roof Rectangular Storage Tank

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ABSTRACT

Sloshing Wave Impact Force (SWIF) caused by liquid motion during seismic excitations is investigated in this paper. When the freeboard is insufficient, the liquid waves collide to the tank roof on which uplift forces are produced. Due to the complication of sloshing impaction, there is no comprehensive investigation that can clarify the various aspect of this phenomenon. Therefore, most of standards don't recommend any method to evaluate SWIF. Alternatively, the main approach of related codes and standards is to suggest a required freeboard in order to prevent collision of sloshing wave to the tank roof instead of evaluating the SWIF. However, suggested freeboard is too high to meet economic considerations in some cases. Therefore, the impact forces should be reasonably evaluated based on the experimental measurements and analytical solutions. An experimental investigation has been implemented to clarify the influence of various geometrical parameters on the impact roof pressure and force values of a rectangular tank. A series of shaking table tests are conducted for a partially filled rectangular tank under harmonic and different earthquake excitations. The experimental measurements for SWIF are compared with those recommended by code provisions and the effects of various parameters on SWIF are discussed.

Keywords:
Sloshing wave force; Freeboard; Experimental measurements; Partially filled rectangular tank; Different excitations

1. Introduction

Fluid sloshing in liquid storage tanks with a free surface is of interest in a variety of engineering fields. It is known that partially-filled tanks with fluids are prone to violent sloshing under certain dynamic conditions. For example, when the frequency of the tank motion is close to the natural frequency of the interaction between the sloshing fluid and the structure, the enhanced fluid motion creates localized high impact loads on the tank walls and ceiling, which can cause structural damage [1]. The convective part of liquid (the upper part which is subjected to the sloshing effects) is the main source of the sloshing wave and needs to slosh freely in the upper part of tanks. Hence, a sufficient freeboard is usually provided to prevent the impaction of liquid wave to tank roofs during earthquakes. However, defining the maximum sloshing wave height and the required freeboard is somehow challenging. On the other hand, providing a large freeboard is not economical, particularly for broad tanks and elevated tanks. Therefore, large-amplitude sloshing in the case of insufficient freeboard may result in significant damage to tank roofs [2].

Liquid sloshing has been attracting much attention over the past few decades. There has been a considerable amount of studies on the LNG vessels in order to attain a better understanding of pressure distribution among the tanks' structure under sea waves excitation. However, the results of studies on LNG tanks are not usable for liquid storage tanks under seismic excitation according to the shapes of tank roofs and the nature of excitation [3-5]. All studies about this phenomenon could be divided into three parts: a) analytical studies, b) numerical approaches, and c) experimental studies.

An analytical model allows the understanding of sloshing mechanics and extensive parametric study. Analytical formulation of liquid equations is well documented by many researchers for tanks with various regular geometries. The general equation of liquid motion in closed containers is often simplified by assuming the container rigid and impermeable. Furthermore, the liquid is assumed ideal which is inviscid, incompressible, and irrotational. Capillary or surface tension effects are generally ignored in analytical formulation of liquid sloshing [6]. Early studies of sloshing focused on linear problems in two dimensional simple geometrical containers, which can be solved by analytical methods [7]. Analytical methods could predict the sloshing wave's motion based on the simplified assumptions for nonviolent motion. However, the analytical predic-tion of violent sloshing characterized by wave overturning and breaking is a difficult task, if not impossible, to obtain [6].

Recent advances in computational methods and computer power make it possible for numerical methods to be applied to study large motion problems of free surface flow. The accuracy of traditional numerical methods, such as finite difference method, boundary element method, finite volume method, and finite element method depends on the size of the applied mesh in simulated model. This feature complicates the process of liquid motion simulation by above-mentioned methods, especially when the free surface is presented. In recent years, a new generation of numerical methods has been developed, i.e. meshless (or mesh-free) methods that outperform conventional mesh-based methods in dealing with discontinuous motion. Meshless methods have been found to have advantages in dealing with large-amplitude free surface flows, moving inter-faces, large motion and complex and deformable boundaries [6].

There are only a limited number of studies on the sloshing impact during earthquakes. Most experimental studies have been carried out in marine industries, like liquefied natural gas carriers. One of the earlier experimental investigations of nonlinear, free-surface standing waves was reported by Taylor [8] who focused on the wave crest in the center of a rectangular tank. Milgram [9] studied the sloshing impact pressure on roofed liquid tanks using a series of experiments. Kobayashi [10] studied the effects of impulsive pressure due to the sloshing impacts. Minowa et al. [11] conducted a series of shaking table tests on a rectangular tank to measure the impact pressure on tank roofs, natural frequencies and modes of bulging vibrations. Wei Chen et al. [12] simulated large-amplitude sloshing motion of liquid subjected to harmonic and earthquake base excitations. It was concluded that non-linear sloshing effects should be considered in seismic-resistant tank design. Hakan Akyildiz [13] investigated pressure variations and three-dimensional effects on liquid sloshing loads in a moving partially filled rectangular tank, numerically and experimentally. Malhotra [14] proposed a simple method of estimat-ing sloshing loads when the freeboard is insufficient. It is noticeable that this method became as the basis of applied forces estimation to the tank roof in regulations. Goudarzi et al. [15] conducted a series of tests to investigate the hydrodynamic sloshing force on the storage tank roofs. They developed an analytical solution to model sloshing impact force on tank roofs. Their analytical solution parameters were calibrated with the experimental measurements [16]. Hakan Akyildiz et al. [17] experimentally investigated the liquid sloshing in a cylindrical tank with various fill levels and ring baffles under the excitations of roll motion. Their primary objectives of the experimental study were to examine the relative effectiveness of various baffle arrangements. They found that the ring baffle arrangements are very effective in reducing the sloshing loads [17]. Jin et al. [18] designed a horizontal perforated plate and incorporated into a rectangular liquid tank that was excited under different amplitudes and frequencies. The experimental results indicated that the horizontal perforated plate can significantly restrain violent resonant sloshing in tanks [18].

In the present study, the effects of various geometrical parameters such as freeboard, liquid height and excitation amplitude on the impact pressure and force values applied to the rectangular tank roof is experimentally investigated. Moreover, the results of available simplified approaches to evaluate the SWIF are compared to the experimental measurements and the validity of this analytical method is discussed, as well.

2. Experiments

This study aims to investigate the sloshing phenomenon using a series of experiments on a rectangular tank. The dimensions of rectangular tank are 100 x 100 x 30 (height x length x width). The tank comprises of plexiglass with thickness of 1.5 cm. The tank walls are supposed to be rigid during the excitations. Tank model and the position of four force transducers and one pressure gauge which are mounted on the roof are shown in Figures (1) and (2), respectively.

The tank is fixed to the table by angle profiles beam placed at the periphery of the tank's top and bottom. The upper angles restrain the shell lateral deformation when the liquid height is high and provide a platform to hold the suspended roof of the tank. The roof is suspended by four rods to attain the net SWIF and provide the ability to change the roof vertical position. A small gap is provided between the roof and shell of the tank in order to prevent the friction forces. As a result, the total force of the tank roof is only caused by the sloshing motion of the contained liquid.

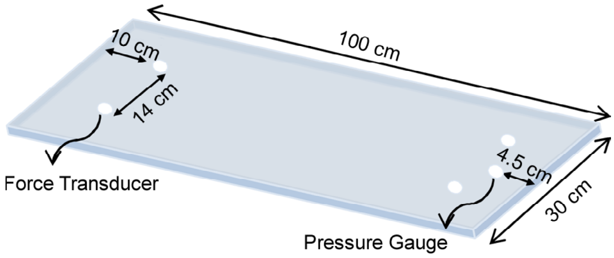


Figure 2. Position of pressure gauge and force transducers.

Nine experiments were conducted in order to consider the effects of freeboard height on pressure and force applied to the roof, where harmonic excitation with 1 cm amplitude has been used. The harmonic excitation of applied displacement time history is as follows:

$$d = A \times Sin\,(\omega_n t) \quad (1)$$

where A is the amplitude and ω_n is the frequency of the excitation. The frequency of harmonic input excitations is set to the primary natural frequency of liquid sloshing motion to intensify the SWIF during the liquid impact. The primary natural frequency (ω_n) of a rectangular tank depends on the tank geometries and obtained from the following equation:

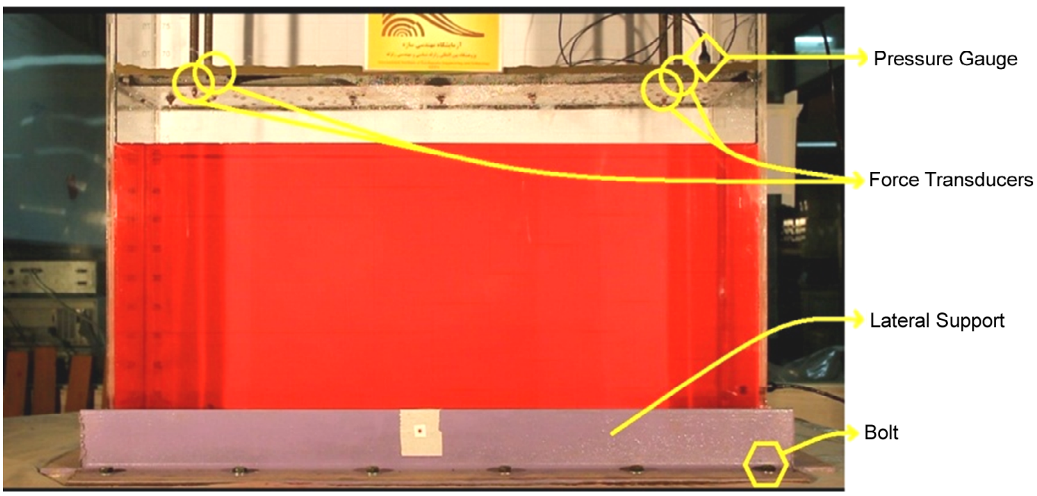


Figure 1. Experimental setup and the small scale tank model.

$$\omega_n(rad/\sec) = \sqrt{\frac{1}{\tanh(\pi H_w/L)}\frac{\pi g}{L}} \qquad (2)$$

where L is the tank length and H_w is the liquid height in the storage tank.

3. Experimental Results

3.1. Influence of Various Geometrical Parameters

As an example, the time histories of SWIF for 50 cm liquid height (H_w= 50 cm) and different freeboards are shown in Figure (3). It could be seen that the force values are increased when the roof is set on higher freeboards height.

The maximum value of sloshing impact forces are extracted from the time history results of shaking table tests and presented in Figure (4). The numerical values of these forces are also tabulated in Table (1). As can be seen, the variation trends of maximum pressure and force values are identical for various liquid heights. As the freeboard height is increased, both values of local pressure and SWIF are amplified. It should be noted that SWIF

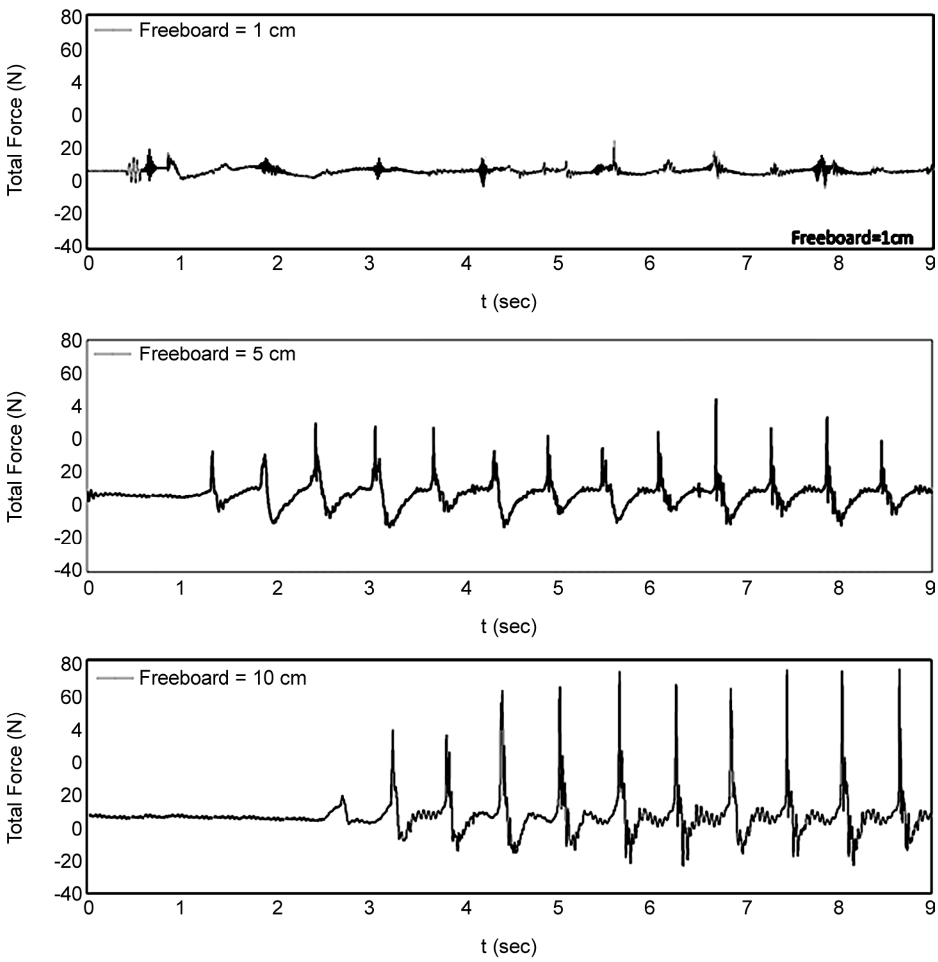
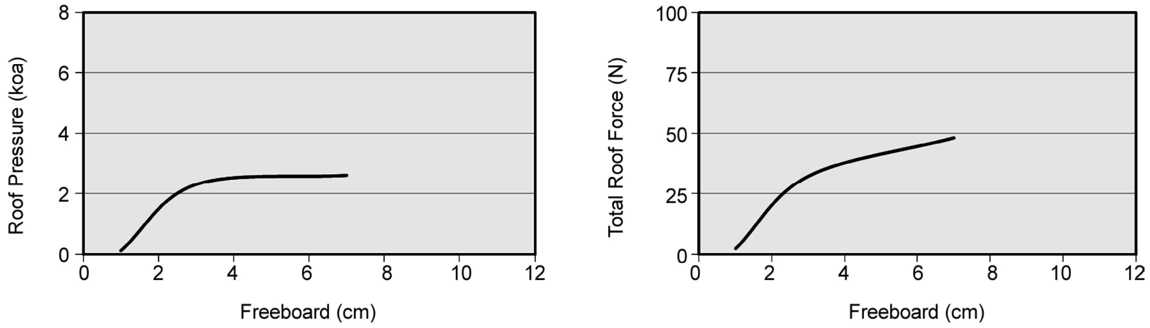


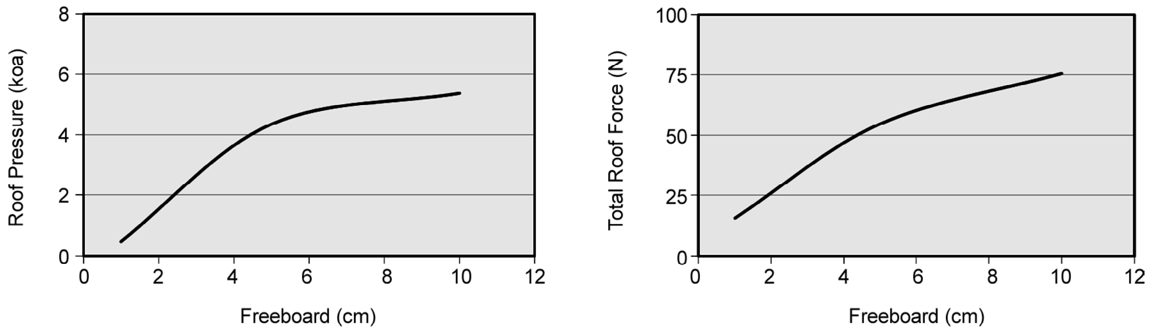
Figure 3. Sample time histories of impact roof force for H_w = 50 cm and different freeboards.

Table 1. Roof pressure and force values under harmonic oscillation with 1 cm amplitude.

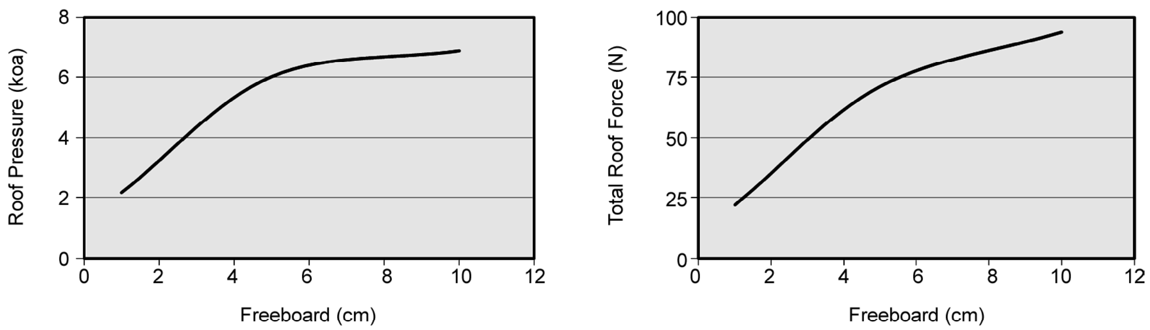
	Roof Pressure (Kpa)				Roof Force (N)		
	Freeboards				Freeboards		
H_w=20cm	1 cm	3 cm	7 cm	H_w=20cm	1 cm	3 cm	7 cm
	0.12	2.3	2.6		2.3	36.3	48.1
H_w=50cm	1 cm	5 cm	10 cm	H_w=50cm	1 cm	5 cm	10 cm
	0.47	3.35	5.38		15.6	49.7	75.5
H_w=70cm	1 cm	5 cm	10 cm	H_w=70cm	1 cm	5 cm	10 cm
	2.17	6.57	6.88		22	73.2	93.8



(a) Roof Pressure and Force Values of Different Freeboard Heights for H_W = 50 cm



(b) Roof Pressure and Force Values of Different Freeboard Heights for H_W = 20 cm



(c) Roof Pressure and Force Values of Different Freeboard Heights for H_W = 70 cm

Figure 4. Acceleration time histories used as the input seismic excitations in experiments.

increment continues up to a specific freeboard height and then decrease. Therefore, there is a risky freeboard height for a certain liquid height to cause maximum liquid impact on the roof. According to the code approaches, SWIF is increased by the reduction of freeboard height. However, the results of the experiments show the adverse trend in respectively small freeboards. It seems that the presence of the tank roof affect the liquid velocity field in small freeboard cases. As a result, the general trend of convective liquid motion is changed and the sloshing waves are damped by the roof before rising to those height predicted by linear potential theory.

Hence, despite of the general belief, not only the reduction of freeboard height does not increase the SWIF, but also it may be an efficient way to reduce the adverse effects of sloshing wave height.

3.2. The Validity of Simplified Method

In order to evaluate the accuracy of simplified method proposed by previous researchers and recommended by the codes, 36 different experiments are conducted including 27 cases with various roof heights and 9 cases without the presence of tank roof to measure the maximum sloshing wave height. These maximum values are required to calculate the SWIF by the simplified approach. For above-mentioned cases, three seismic input excitation including Tabas, Chi-Chi and Kobe earthquake records are selected. The scaled acceleration time histories of these records are shown in Figure (5).

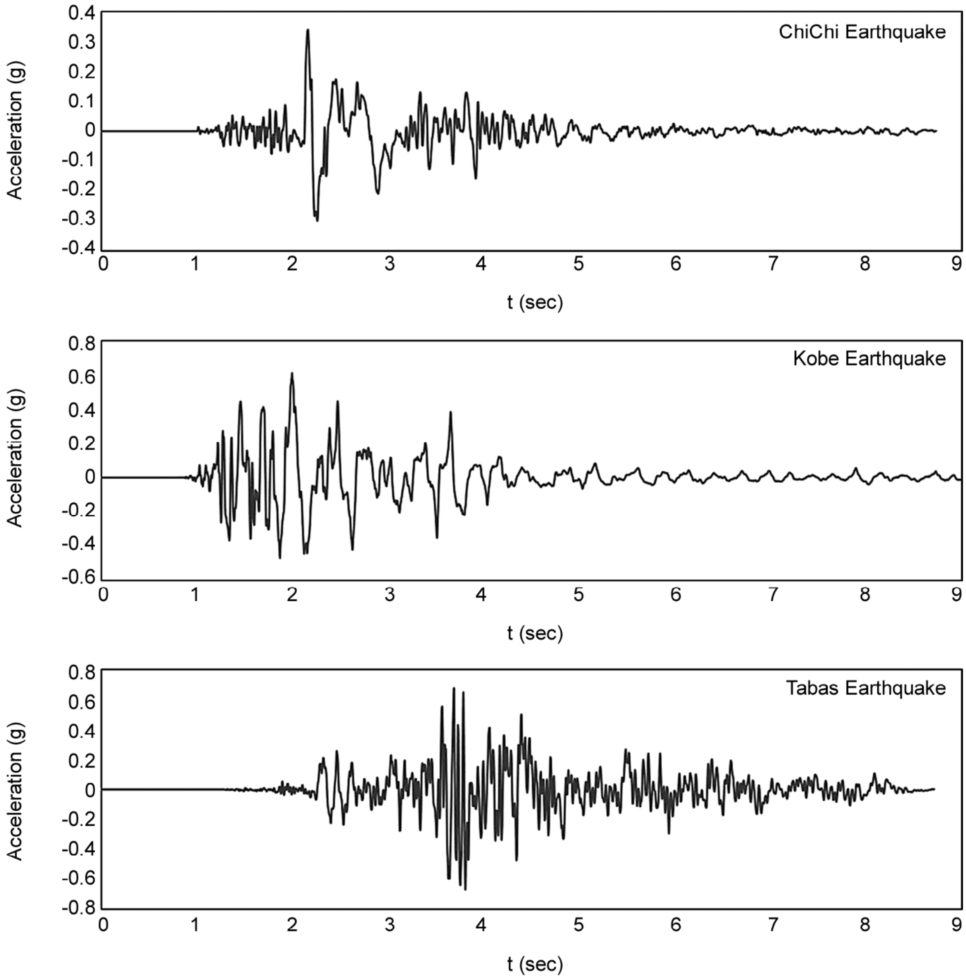


Figure 5. Acceleration time histories used as the input seismic excitations in experiments.

Before discussing the results, simplified analytical method that is used in many regulations of tank design to estimate SWIF is briefly introduced. In this method, the hydrostatic pressure of liquid waves is considered to calculate the SWIF. For this purpose, the portion of the fluid volume above the ceiling is considered as computational volume for the estimation of SWIF. Considering the parameters illustrated in Figure (6), the SWIF could be estimated based on the wetted surface length and maximum sloshing height of un-roofed tank.

By equating the volume of empty reservoir before and during the excitation, we will have:

$$d_f L = \frac{1}{2} y(L - X_f) \tag{3}$$

Obtaining the value of y from the angle of θ and replacing it in the equation above, wetted surface length (X_f) is achieved.

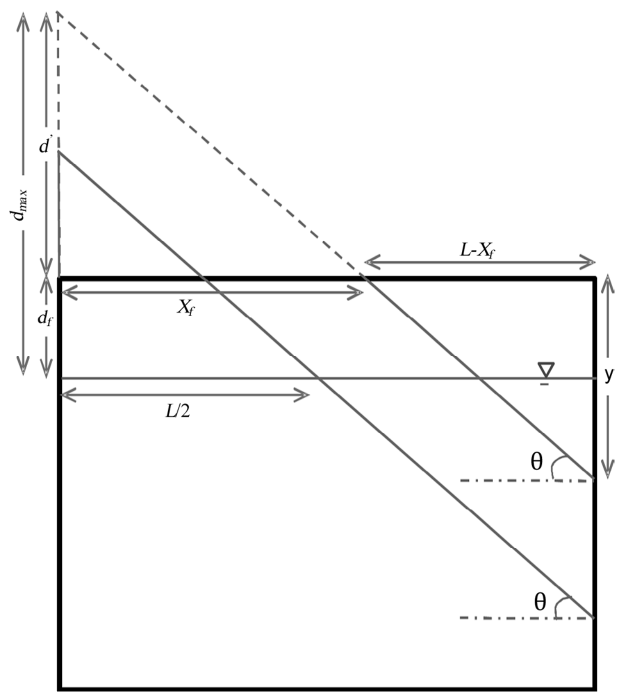


Figure 6. The schematic view of the simplified method assumptions.

$$\left.\begin{array}{l}\tan\theta = \dfrac{y}{L - X_f} = \dfrac{a_x}{g} \rightarrow y = \dfrac{a_x}{g} \times (L - X_f) \\ \tan\theta = \dfrac{a_x}{g} = \dfrac{d_{\max}}{0.5L}\end{array}\right\} \rightarrow$$

$$y = \frac{d_{\max}}{0.5L}(L - X_f) \rightarrow$$

$$d_f L = \frac{(L - X_f)^2}{L} d_{\max} \rightarrow \qquad (4)$$

$$L - X_f = \sqrt{\frac{d_f L^2}{d}} \rightarrow X_f = L(1 - \sqrt{\frac{d_f}{d}})$$

Having the length of wetted surface and maximum vertical displacement, the total applied force to the ceiling could be obtained by the following equation:

$$F_{\max} = \frac{1}{2} \times P_{\max} \times X_f \times b = \frac{1}{2} \times \gamma \times d' \times X_f \times b \qquad (5)$$

where $F_{\max}$ is the maximum force applied to the tank roof, γ is the specific weight of water, d' is the maximum virtual vertical displacement of liquid, X_f is the wetted surface length and b is the width of the rectangular reservoir.

In order to have the maximum free surface displacement in un-roofed tanks, several experiments were conducted under earthquake excitement for the liquid heights of 20, 50 and 70 cm. These maximum free surface displacement ($d_{\max}$) values obtained from these experiments are extracted and tabulated in Table (2).

θ angles that is slope angle of the free surface is identical with and without presence of tank roof. Hence, $d_{\max}$ to d' ratio is obtained as follows.

$$\left.\begin{array}{l}\tan\theta = \dfrac{d_{\max}}{L/2} \\ \tan\theta = \dfrac{d'}{X_f}\end{array}\right. \rightarrow d' = \frac{d_{\max} \times X_f}{L/2} \qquad (6)$$

Substituting d' from Eq. (6) into Eq. (5), the maximum force applied to the roof is rewritten as follows:

$$F_{\max} = \frac{1}{2} \times \rho g \times \frac{d_{\max} \times X_f}{L/2} \times X_f \times b = \frac{\rho g b}{L} \times d_{\max} \times X_f^2 \qquad (7)$$

The estimated SWIF results calculated by Eq. (7) are compared with the experimental measurements and presented in Figure (7). As can be seen, the simplified method results are based on the assumption that the SWIF is increased by the reduction of freeboard height. However, the experimental measurements reveal opposite trend as it was observed for the same cases under harmonic excitations. In other words, the smaller freeboards do not necessarily lead to higher SWIF based on the experimental results. It seems that the sloshing wave height lose the potential to experience its maximum virtual value which is calculated from the potential theory when there is no tank roof. In fact, the regime of the fluid flow inside the tank is considerably affected by the presence of the tank roof in the case of small freeboard height. This point is ignored in the formulation of simplified method in which the maximum virtual sloshing height is directly obtained from the analytical solution of un-roofed tanks. This fact is more highlighted when the difference between the simplified and experimental results is considered as presented in Table (3). It is clear that the differences are

Table 2. Maximum vertical displacement of water in the upper end of tank (d_{max}).

Liquid Heights	EQ	Maximum Free Surface Displacement (d_{max})
H_w = 20 cm	Chichi	7 cm
	Kobe	11 cm
	Tabas	6 cm
H_w = 50 cm	Chichi	11 cm
	Kobe	18 cm
	Tabas	12 cm
H_w = 70 cm	Chichi	12 cm
	Kobe	21 cm
	Tabas	15 cm

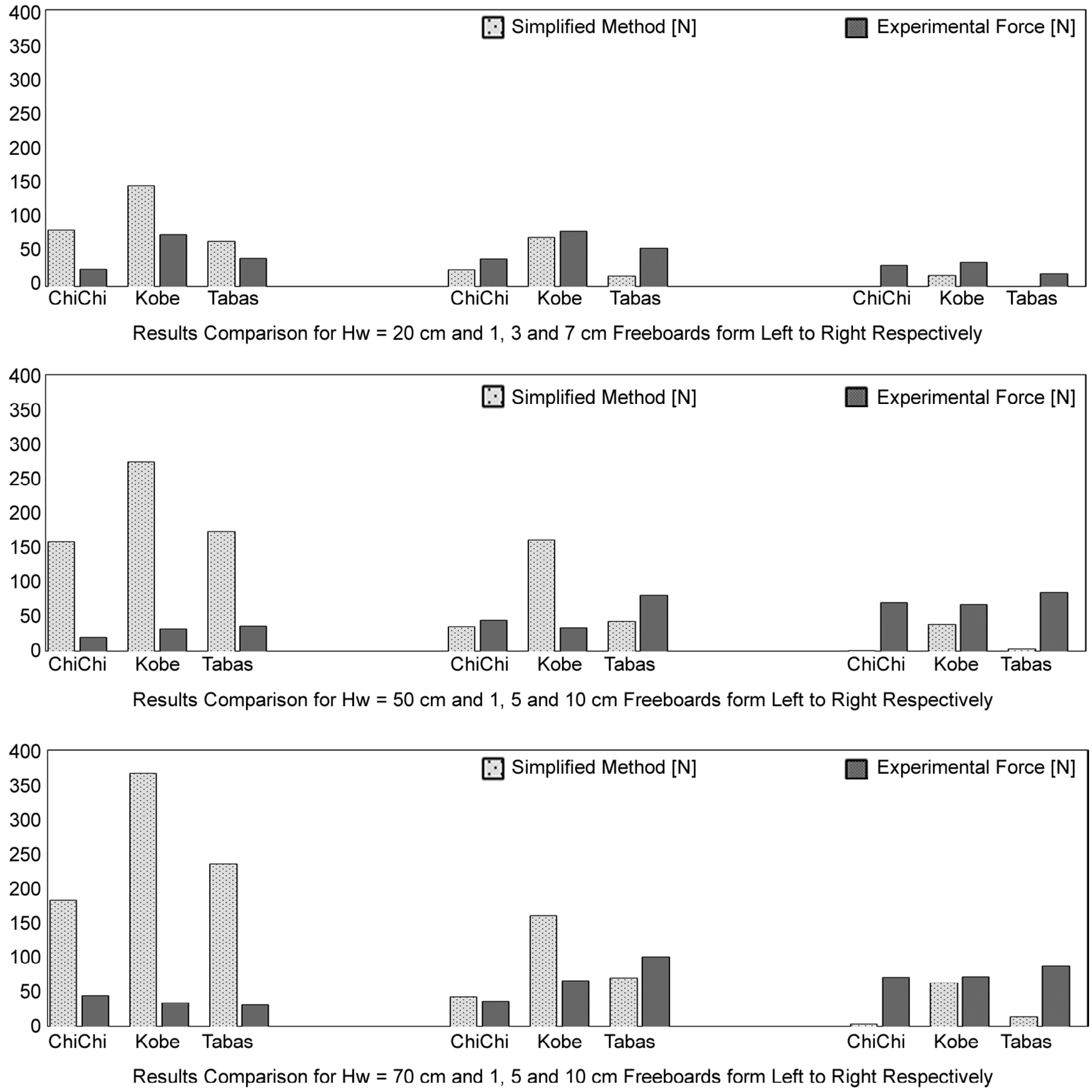


Figure 7. Comparison of analytical solution and experimental measurments.

Table 3. Percentage deviation of experimental measurements and simplified method.

	$H_w = 20$ cm								
	$Fr = 1$ cm			$Fr = 3$ cm			$Fr = 7$ cm		
	Exp	**Simp**	**Deviation (%)**	**Exp**	**Simp**	**Deviation (%)**	**Exp**	**Simp**	**Deviation (%)**
Chi-Chi	25	81.7	-226%	40	23.8	40.5%	30	0	100%
Kobe	75	145.2	-94%	80	71.5	10.6%	35	15.7	55.1%
Tabas	41	65.7	-60%	55	14.8	73.1%	18	0	100%
	$H_w = 50$ cm								
	$Fr = 1$ cm			$Fr = 5$ cm			$Fr = 10$ cm		
	Exp	**Simp**	**Deviation (%)**	**Exp**	**Simp**	**Deviation (%)**	**Exp**	**Simp**	**Deviation (%)**
Chi-Chi	20.2	158.7	-685%	44.6	35.2	21%	70.2	0.8	98.9%
Kobe	32.1	274.5	-755%	33.3	160.7	-382%	67.1	38.6	42.5%
Tabas	35.8	172.9	-382%	80.6	43.2	46.4%	84.8	2.8	96.7%
	$H_w = 70$ cm								
	$Fr = 1$ cm			$Fr = 5$ cm			$Fr = 10$ cm		
	Exp	**Simp**	**Deviation (%)**	**Exp**	**Simp**	**Deviation (%)**	**Exp**	**Simp**	**Deviation (%)**
Chi-Chi	44.8	183	-308%	36.5	43.2	-18.3%	70.9	2.8	96%
Kobe	34.3	366.3	-967%	66.5	160.7	-141%	72.1	63.3	12.2%
Tabas	30.8	235.2	-663%	100.5	70.6	29.7%	86.9	14.3	83.5%

extensively enhanced in smaller freeboard heights. Besides, the general trend of simplified method prediction is not acceptable for the cases considered in the experiments.

4. Conclusion

Sloshing Wave Impact Force (SWIF) caused by liquid motion during seismic excitations is investigated in this paper. A series of shaking table tests are conducted for a partially filled rectangular tank under harmonic and different earthquake excitations. Forty five experiments are conducted in the laboratory of international institute of earthquake engineering and seismology where nine experiments are associated with the investigations of freeboard height effects and the rest are performed to evaluate the accuracy of simplified method recommended by code provisions. The following conclusions can be made:

- The general variation trends of maximum pressure and force values are identical for various liquid heights. As the freeboard height is increased, both values of local pressure and SWIF are increased. Therefore, there is a risky freeboard height for a certain liquid height to cause maximum liquid impact on the roof.
- The smaller freeboards do not necessarily lead to higher SWIF based on experimental results. This is due to the fact that the regime of the fluid flow inside the tank is considerably affected by the presence of the tank roof in the case of small freeboard height. This point is ignored in the formulation of simplified method recommended by the codes. Hence, despite of general belief, not only the reduction of freeboard height does not increase the SWIF, but also it may be an efficient way to reduce the adverse effects of sloshing wave height.
- The general trend of simplified method prediction is not acceptable for the tanks excited under seismic loads.

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