Application of Reliability in Stability Analysis of an Earth Dam

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ABSTRACT

In geotechnical problems, uncertainty and inherent variability in soil strength parameters is undeniable. A common and effective way of considering uncertainty is Monte Carlo simulation method. In this method, the primary objective is to obtain the reliability index and probability of failure for the critical slip surface, or in other words, deterministic slip surface. However, the slip surface with the minimum factor of safety may not be the surface with minimum reliability index or the highest probability of failure. In the present research work, this problem will be scrutinized on an earth Dam. Furthermore, the global reliability index and probability of failure of the dam will be introduced. Specific gravity, seismic pseudo static coefficient and soil strength parameters including cohesion and friction angle are taken into account as the sources of uncertainty. The results show that the common approach of calculating the reliability index may not be conservative.

Keywords:
Reliability analysis; Stability analysis; Earth dam; Reliability index; Global reliability index

1. Introduction

There are several uncertainties in slope stability problem such as natural variability of soil properties, simplifications and estimation in geotechnical models. Depending on the degree of these uncertainties, probability of slope failure might be considerable. Several probabilistic methodologies have been developed to utilize these uncertainties in slope stability analysis, such as the First Order Second Moment (FOSM) method [1-3], First Order Reliability Method (FORM) [4-6] and Monte Carlo Simulation (MCS) method [7-11]. Among these methods, Monte Carlo Simulation method is a popular technique to evaluate probability of slope failure. MCS method deals with different deterministic analysis method such as finite element methods and limit equilibrium methods.

Limit equilibrium method is one of the most popular techniques of assessing the stability of embankment dams. Calculations usually consist of computing a factor of safety. In this approach, the slope is analyzed by assuming a slip surface whose factor of safety will be calculated by dividing the forces resisting movement by the forces driving movement. Calculating procedure of the factor of safety will be applied on different slip surfaces and finally the minimum factor of safety will be specified. If this factor is above the acceptable limit, the dam is deemed stable (this method is called deterministic analysis). The most common limit equilibrium techniques are methods of slices where soil mass is divided into vertical slices, Figure (1). Janbu [12], Bishop [13], Morgenstern [14] and Spencer [15] are examples of the slices method. In earthquake-prone areas, the analysis is performed under static and pseudo-static conditions.

Critical states in the slope stability control of

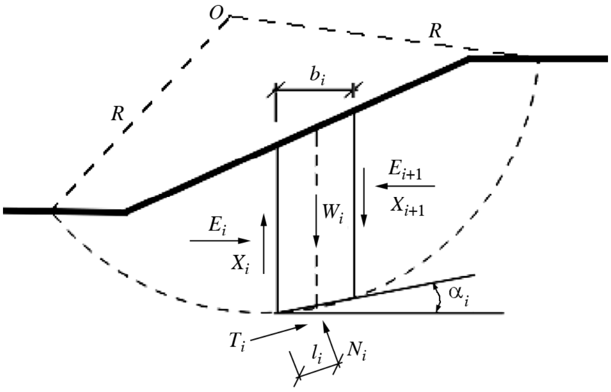

Figure 1. Method of slices.

dams are classified as follows:

1) All phases of construction
2) End of construction condition
3) Rapid drawdown
4) Steady state seepage
5) Earthquake in all of the above conditions

Federal Energy Regulatory Commission (FERC) suggested minimum factors of safety as shown in Table (1). The accepted factor of safety actually depends on the uncertainty involving valid engineering data.

The factor of safety calculated by the common method of dam design is often applied to conditions that involve wide varying degrees of uncertainty; moreover, dam design is a rather complex issue. The standpoint of factor of safety is a correct but simplistic view. The source of uncertainty and the complexities in dam design will be scrutinized bellow.

The main causes of complexities in dam design are as follows [17]:

- All dams are different in terms of geometry and the construction materials.
- The foundation is an important part of the structure and needs careful exploration and improvement.
- The local seismic risk might modify the design.
- Almost all materials used for dams are local: their characteristics have to be identified and improved upon each dam's design based upon the optimized utilization of its materials, the possible construction methods, and the available equipment to transport and improve millions of tons of various materials.

Natural variability is a source of uncertainty in soil properties; the other important source is limited knowledge. Increasingly, these are referred to as aleatory and epistemic uncertainty, respectively. Limited knowledge usually causes systematic errors. For example, limited numbers of tests lead to statistical errors in estimating a mean trend, and if there is an error in average soil strength it does not average out. In geotechnical reliability, the most common sources of knowledge uncertainty are model and parameter selection [18]. Aleatoric uncertainties are typically modeled using random variables with given probability density function [19]. The purpose of Monte Carlo simulation technique is to propagate aleatoric and epistemic uncertainties related to risk [20].

As it was mentioned, one of the most common methods of reliability evaluation is Monte Carlo simulation method. Generally, in reliability analysis of slope stability problems using MCS, the reliability index (β) and probability of failure (p_f) are calculated for the critical slip surface. The β and p_f will be later explained in detail. Critical slip surface is the surface with minimum factor of safety (F.S) from the deterministic analysis approach. It means that in classical uncertainty analysis of slopes, the first step

Table 1. FERC minimum factors of safety [16].

Loading Condition	Minimum Factor of Safety	Slope to be Analyzed
End of construction	1.3	Upstream and downstream
Rapid drawdown	>1.1	Upstream
Rapid drawdown from spillway crest or top of gates	1.2	Upstream
Steady seepage with maximum storage pool	1.5	Upstream and downstream
Steady seepage	1.4	Downstream
Earthquake (for steady state seepage conditions with seismic loading using the seismic coefficient method)	>1	Upstream and downstream
Earthquake (for all dynamic analyses using a deformation method)	<2	Feet of Newmark-type deformation along the potential failure plane

is to define the critical slip surface with minimum F.S, and then β and p_f are obtained for this specific slip surface. The important aspect of slope probability evaluation, neglected in the mentioned approach, is that the failure surface with minimum F.S does not necessarily involve the highest probability of failure or the lowest reliability index. Therefore, the first part of the present study focuses on the reliability analysis of an earth dam using MCS and on a comparison between the failure surfaces corresponding to the minimum value of reliability index and the one corresponding to the minimum factor of safety.

The second problematic condition of this approach is that the focus of the reliability analysis is on a unique slip surface. In other words, the result of this analysis will be dependent on the failure surface. If the failure surface changes, the results of the reliability analysis will change; hence, the next part of the study focuses on independency of the reliability analysis results from the slip surface. At the end, this subject would also be discussed in detail.

3. Probability basic concepts

The most useful probability concepts used in this study are as follows:

Coefficient of variation: The *COV* represents a relative and dimensionless measure of dispersion and is expressed as:

$$COV = \frac{\sigma_x}{\mu_x} \times 100\% \tag{1}$$

The *COV* has been commonly used to describe the variation of many geotechnical soil properties and insitu test parameters. Note that the mean, standard deviation, and *COV* are interdependent - knowing any two will give the third. In practice, by assuming that the *COV* is similar to previously measured values from other data sets of the same parameter, it is convenient to estimate *COV* of the geotechnical soil parameters where little data are available. A summary of the average *COV* values reported in the literature is presented in Table (2) [21].

Table 2. The average of COV reported in literature [21].

Soil Properties	Cohesion	Specific Gravity	Friction Angle
COV (%)	40	7.1	12.6

Probability distribution: The Probability Density Function (PDF) is a function that assigns a probability to every interval of the outcome set for random variables [21]. A normal probability density function is illustrated in Figure (2). The area under the PDF is always unity.

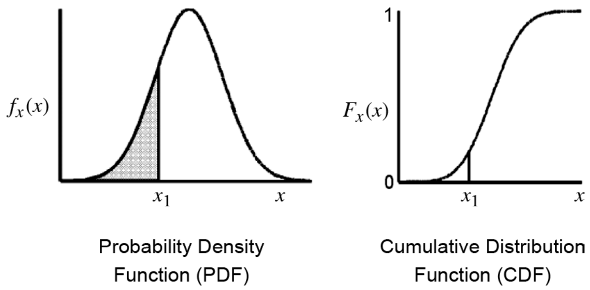

Figure 2. Normal probability density function and cumulative distribution function.

An alternative way of presenting the same information is in the form of a cumulative distribution function (CDF), which gives the probability that the variable will have a value less than or equal to the selected value. The CDF is the integral of the corresponding probability density function.

The normal or Gaussian distribution is the most common type of probability distribution function and the distributions of many random variables conform to this distribution. It is generally used for probabilistic studies in geotechnical engineering unless there are good reasons for selecting a different distribution [24-25].

Reliability index: This parameter is often used to express the degree of uncertainty in the calculated factor of safety. According to the definition, reliability index is defined as [22]:

$$\beta = \frac{E(F) - 1.0}{\sigma(F)} \tag{2}$$

where β is the reliability index, $E(F)$ the expected value of the factor of safety, and (F) is the standard deviation. β describes the stability by the number of standard deviation separating the mean factor of safety from its defined failure value of 1.0. It can also be considered as a way of normalizing the factor of safety with respect to its uncertainty [23]. The reliability index definition is shown in Figure (3).

Probability of failure: Another way of looking

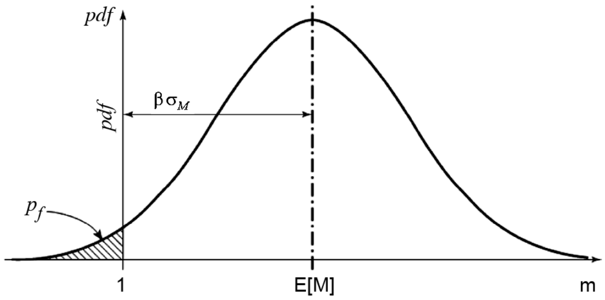

Figure 3. Definition of reliability index (β).

the risk of instability is what is known as probability of failure (P_f). P_f is determined by counting the number of factor of safety below 1.0 and then taking this number as percentage of the total number of Mont Carlo Trials. For example, if there are 1000 Monte Carlo trial and 100 of them are below 1.0, then the probability of failure is 10% [23].

The reliability indices for most geotechnical components and systems lie between 1 and 5, corresponding to probabilities of failure ranging from about 0.16 to 3×10^{-7} as shown in Figure (4) [18].

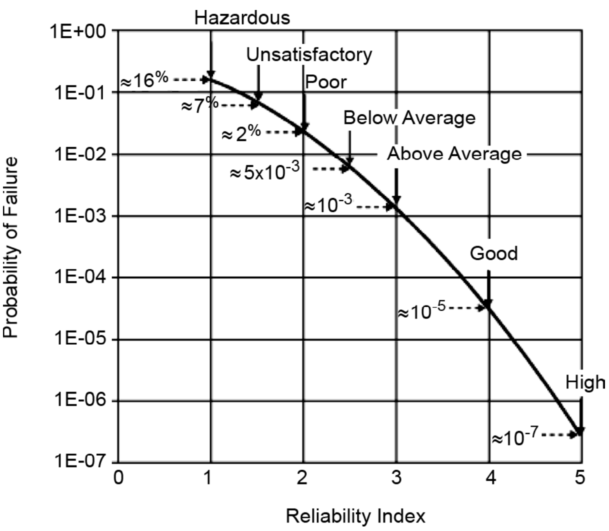

Figure 4. Relation between probability and reliability index [18].

4. Common Procedure of Monte Carlo Simulation Method (MCS)

MCS method is a simple but versatile computational procedure for considering the uncertainties involving the slope stability problem. In this research work, for uncertainty analysis, MSC method has been used. The procedure of MSC method for probability evaluation of dams involves the following steps:

1. The stability of dam will be perused using a deterministic method with GEO-SLOPE/W. In this study, Morgenstern-Price method was used, and the critical factor of safety and its corresponding failure surface were obtained for the predefined loading condition.

2. Choosing input parameters with considerable uncertainty. Soil strength parameters including *c* and ϕ, specific gravity and seismic pseudo static coefficient (k_h) are taken into account as input parameters with noticeable uncertainty.

3. Representing variability of input parameters in term of a selected distribution model; so the parameters with uncertainty will be quantified. Normal probability density function was used in this step.

4. Random sampling of the input parameters. In this step, using the probability density function of the previous step, random sampling of all the input parameters was taken into account and 2000 random samples for every input parameter were achieved.

5. For all random samples of the previous step, factors of safety of dam for specific slip surface obtaining from step 1 were calculated using a deterministic method. Two thousands of random samples result in 2000 safety factors. It should be noted that in the calculation of factor of safety, slip surface was supposed to be constant.

6. Regarding the results of step 5 and through definition of probability of failure, P_f is achieved.

7. The probability density functions of factor of safety was drawn based on the results of step 5.

8. Average and standard deviation of the factor of safety are calculated using the probability density function.

9. Reliability index is calculated using Eq. (4).

5. General Specifications of a Typical Large Dam

The dam is located in the southwest of Iran. It is a 180 m high rockfill dam with central clay, which is classified as a large dam according to ICOLD guideline. Based on the seismotectonic studies, the supposed dam is located in the Zagros fold and thrust belt, the deformation of which appears to be concentrated on basement thrusts and a few transverse strike-slip faults. Zagros is the most seismically active region in Iran. In this region, the activity rate of earthquakes is very high, though most of them have small medium magnitude and occur at a shallow focal depth. In order to estimate the ground motion parameters for the project, the probabilistic, deterministic and statistical methods are used. Seismic design level for dam is defined as Design Basis Level (DBL), Maximum Design Level (MDL) and Maximum Credible Level (MCL). For each of these design levels, the peak ground accelerations are estimated as shown in Table (3).

Figure (5) illustrates the typical cross section of the dam. The properties of the soil are presented in Table (4).

Table 3. Peak Ground Acceleration (PGA) for different design level.

Ground Motion Design Level	Horizontal Component 50%	Horizontal Component 84%
DBL	0. 156 g	0. 27 g
MCL	0. 480 g	0. 768 g

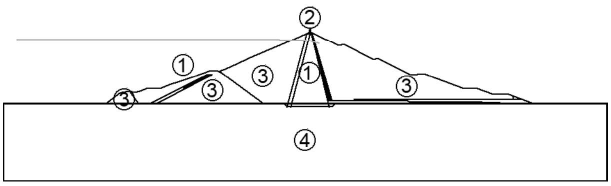

Figure 5. Typical cross section of the dam.

Table 4. Soil Properties of the dam.

Zone	1	2	3	4
Specific Gravity (kN/m^3)	20.5	20.5	23	24
Cohesion (kPa)	20	0	0	210
Friction Angle (deg)	26	30	40	26

6. Reliability Analysis of the Dam

Procedure of MSC method for probability evaluation of dams has been described in section 4. In this part, the MSC method was applied for reliability analysis of the dam. As mentioned in section 2, cohesion, friction angle, specific gravity and also seismic pseudo static coefficient (k_h) are taken into account as input parameters.

In the next step, parameters with uncertainty should be quantified. Using the *COV* cited in Table (2) accompanied by the mean values in Table (3) and utilizing Eq. (3) simultaneously, the standard deviation of cohesion, friction angle and specific gravity were attained; hence, the normal probability density functions for all mentioned parameters were achieved. Table (3) was applied to calculate k_h. As seen in Figure (6), by employing the horizontal component of Table (3), standard deviation can be achieved by the following Equations.

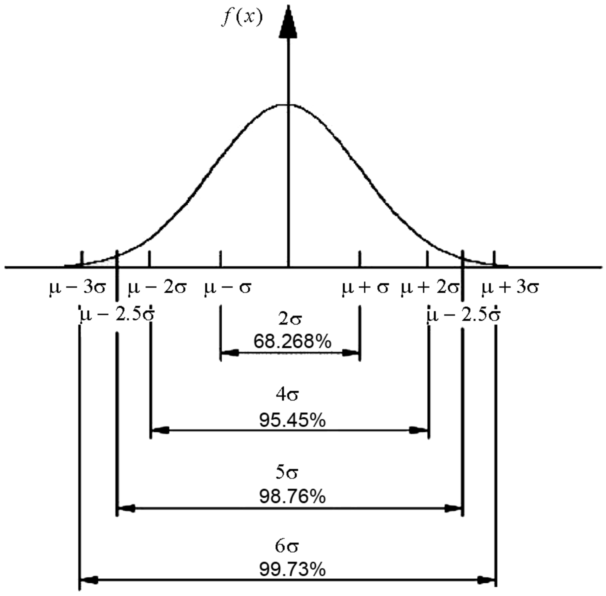

Figure 6. Normal probability density function.

For long-term condition, the k_h is considered as the maximum value of ($0.5 \times \text{PGA}_{\text{DBL}}$) or ($0.33 \times \text{PGA}_{\text{MCL}}$) and for short-term condition, k_h is ($0.33 \times \text{PGA}_{\text{DBL}}$) [15].

After quantifying the parameters, according to the aforementioned steps in part 4, reliability analysis was carried out. For two different loading conditions, the results are as follows:

6.1. Steady State

The results for this specific condition are shown in Figure (7) and Table (5). It can be seen that, the FOS (Factor of Safety) in the common way of dam design is 2.27 for Morgenstern-Price Method; therefore, the dam is safe (refer to the allowable values in Table (1)). The critical slip surface is shown in Figure (7). The result of MSC method for this specific slip surface shows that β is equal to 4.3. Shown in Figure (4), the reliability index greater than 4 cited the level of good from the view point of reliability analysis.

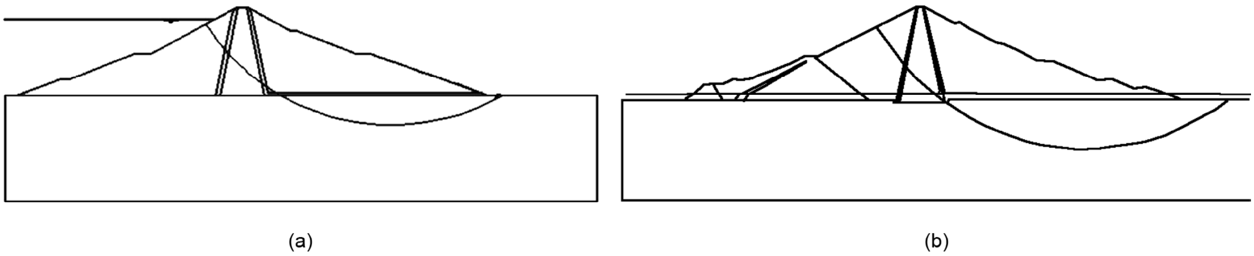

Figure 7. Critical slip surface a) steady state, b) Pseudo-static condition.

Table 5. Results of reliability analysis of the dam.

Method		Steady State	Pseudo-Static Condition
		Morgenstern-Price	Morgenstern-Price
FOS in Common Way of Slope Stability Design		2.273	1.315
MSC method	β	4.3	0.967
	P_f (%)	0	16.15
	random Seed	2000	2000

6.2. Pseudo-Static Condition

The result is presented in Table (5). For this condition, the FOS is 1.315 while β is 0.967 and P_f is 16.15%. According to the minimum factor of safety mentioned in Table (1), the dam is completely safe from the viewpoint of limit equilibrium method, but from the perspective of reliability analysis, as shown in Figure (4), the level of hazard in dam shows that the dam is not in a good condition.

7. Determining the Minimum Reliability Index of the Dam

In the common way of slope probability evaluation, the main goal is to determine the reliability of critical failure surface. In this section, the reliability of other slip surfaces are evaluated and the minimum reliability index is identified as the critical slip surface from the probability point of view. In order to find the failure surface with the minimum reliability index, the following steps were considered.

1. A large number of failure surfaces were supposed and the factor of safety corresponding to each one was determined.

2. The safety factors were sorted from smallest to largest. The number of surfaces considered in this study was 1000.

3. For each of the failure surfaces from step 2, reliability analysis was performed using MCS method. Thus, 1000 reliability indexes and probability of failures were obtained; the minimum reliability index was chosen as the critical slip surface in reliability analysis.

For two loading conditions, the results are as follows:

7.1. Steady State

The results are given in Table (6). Critical reliability index is related to the 72th failure surface and its value is 3.601, while the value of reliability index related to critical slip surface is 4.301. With reference to Figure (5), the reliability of 3.601 and 4.301 show the level of above average and good, respectively. The difference between these two values is considerable. Figure (8) shows five critical slip surfaces and the 74th slip surface. It is obvious that the 74th slip surface is completely different from the others. Therefore, it might be concluded that: Considering the reliability of the critical slip surface is not a relevant criterion for evaluating the probability of the dam.

7.2. Pseudo-Static Condition

For this loading condition, results are presented in Table (6) and Figure (9). The second slip surface has the minimum reliability index and P_f with the value of 0.967 and 16.33%, respectively. The β and Pf for the critical slip surface is 0.967 and 16.15%. Values are rather close and show the same level of

Table 6. Results of MCS on different slip surfaces.

Steady State and Static Condition				Steady State and Pseudo-Static Condition			
Number of Slip Surface	Factor of Safety	Reliability Index	Probability of Failure (%)	Number of Slip Surface	Factor of Safety	Reliability Index	Probability of Failure (%)
1	2.273	4.301	0	1	1.315	0.967	16.15
2	2.274	4.293	0	2	1.322	0.962	16.33
3	2.276	4.172	0	3	1.323	0.992	15.34
4	2.277	4.43	0	4	1.324	0.982	15.57
5	2.277	2.534	0	5	1.325	1.014	14.6
6	2.278	4.184	0	6	1.334	0.991	15.26
7	2.28	4.305	0	7	1.335	0.986	15.46
8	2.282	4.28	0	8	1.336	1.034	13.64
9	2.285	4.401	0	9	1.336	0.97	15.99
10	2.286	4.395	0	10	1.339	1.033	13.86
11	2.287	4.197	0	11	1.341	1.057	13.2
12	2.293	4.541	0	12	1.341	1.025	13.83
13	2.293	4.188	0	13	1.342	1.067	13.1
14	2.295	4.5	0	14	1.343	1.018	14.19
15	2.296	4.339	0	15	1.344	1.041	13.35
16	2.296	4.354	0	16	1.344	1.067	13.04
17	2.296	4.311	0	17	1.345	1.029	13.47
18	2.297	4.206	0	18	1.346	1.05	13.4
19	2.301	4.633	0	19	1.346	1.045	13.27
20	2.302	4.118	0	20	1.346	1.071	13.03
73	2.367	3.863	0	.	.	.	.
74	2.367	3.601	0	.	.	.	.
75	2.368	3.929	0	.	.	.	.
85	2.376	3.612	0	998	1.521	1.354	4.95
89	2.381	3.634	0	999	1.521	1.534	3.29
91	2.383	3.609	0	1000	1.521	1.496	4
96	2.386	3.888	0	-	-	-	-
97	2.386	3.659	0	-	-	-	-
101	2.391	3.684	0				
105	2.394	3.761	0	-	-	-	-
109	2.395	3.712	0				
998	3.281	5.507	0	-	-	-	-
999	3.281	4.897	0	-	-	-	-
1000	3.284	4.589	0	-	-	-	-

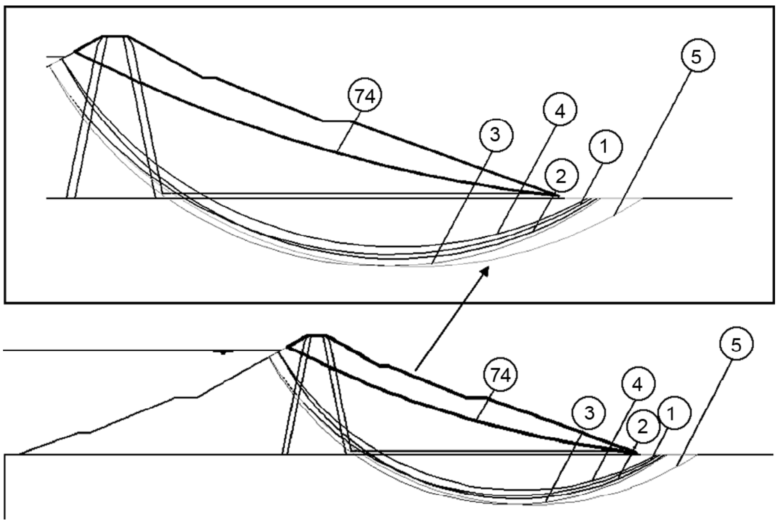

Figure 8. Critical slip surfaces of steady state and static condition.

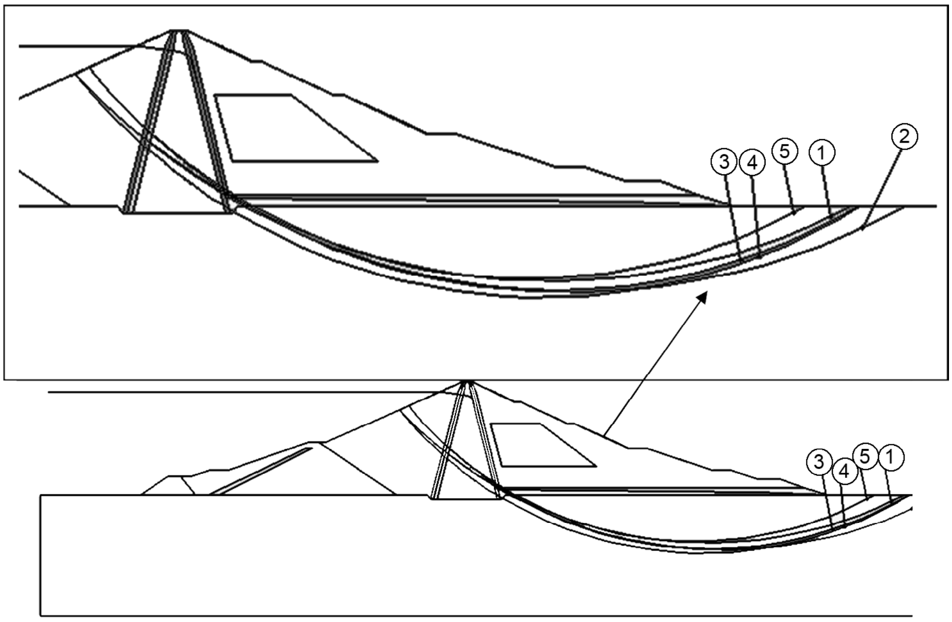

Figure 9. Critical slip surfaces of steady state and pseudo-static condition.

hazard (referred to Figure (5)), but the failure surfaces are a bit different.

8. Estimating the Global Reliability Index and Probability of Failure for the Dam

The missing point for considering the uncertainties in the common approach of MCS and even in the method presented in part 7 is that while the value of input parameters change in every MCS trial, the specific failure surface may not remain as the critical slip surface. In other words, when the input parameters change, the failure surface may change, but the common approach of MCS carry out all the analysis on a fixed failure surface. To overcome this problem, a factor of safety with specific critical failure surface should be calculated for every group of random samples. Therefore, changing the input parameters will also change the critical failure surface. In this section, the extracted reliability index and probability of failure would be called the global β and P_f. In order to extract the global β and P_f, the following steps are performed:

1. After identifying the uncertainty source (C, ϕ, specific gravity and k_h), random sampling is implemented.

2. For each group of random samples of the previous step, the deterministic factor of safety of the dam is calculated using a deterministic method. For this part, Morgenstern-Price Method is used. For each group of random samples, a specific factor of safety is extracted. It seems obvious that for each group of random samples, the critical failure surface is different from the others.

3. Drawing the PDF and CDF of the factor of safety.

4. Average and standard deviation of the safety factors are calculated using probability density function.

5. Reliability index is calculated using Eq. (4). For pseudo-static condition, the results are as follows.

8.1. Steady State and Pseudo-Static Condition

As the first trial, 100 random samples have been selected. The PDF and CDF are shown in Figures (10) and (11). The results for 200, 300, 400 and 500 seed numbers are shown in Figure (12) and Table (6). The results are compared with the common approach of reliability analysis, Table (7) and Figure (12).

In this Figure, the effect of seed random number generator on the reliability index is investigated. For this purpose, several computer runs are conducted by which the seed random number generator is allowed to vary from 100 to 500. For each run, the reliability index and probability of failure are determined. As it can be seen in the diagrams, β and P_f are not sensitive when the seed number is greater than 300. Figure (12) shows that the global β and P_f have higher values compared to the common approach of MCS. The difference is also considerable.

When the seed number is 500, the β and P_f are 1.089 and 14.86%, respectively for the common approach of MCS, while the global β and P_f are 0.89 and 17.4%. Based on Figure (4), the reliability of 1.089 and 0.89 show the levels of unsatisfactory and hazardous, respectively. In other words, the reliability index obtained in common approach is not conservative for design.

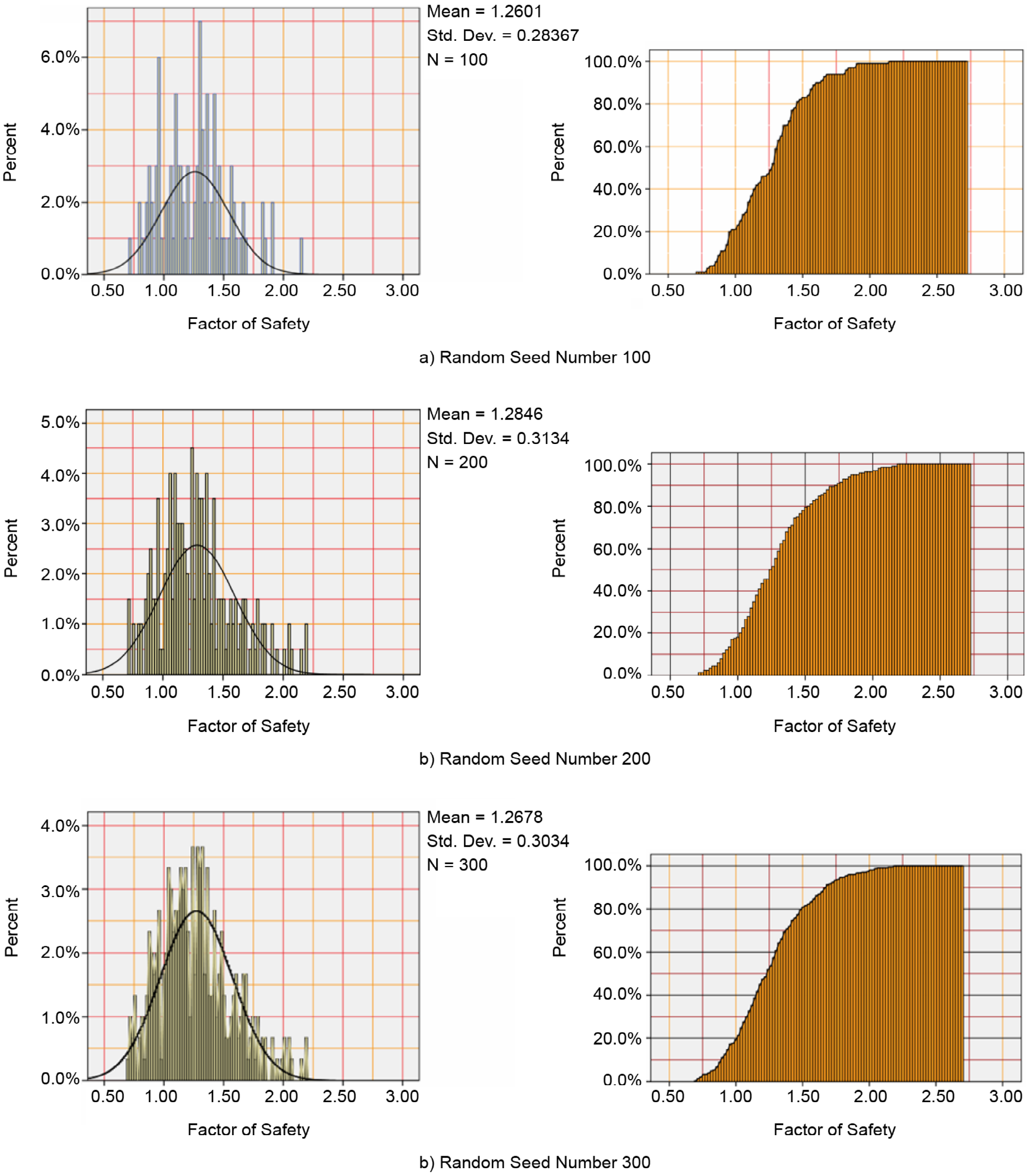

Figure 10. The PDF and CDF of factor of safety for steady state and pseudo-static condition for random seed number between 100 to 300.

Table 7. Results of MCS on different slip surfaces.

Seed Number	100	200	300	400	500
Average Factor of Safety	1.260	1.285	1.268	1.268	1.273
Reliability Index	0.917	0.908	0.883	0.909	0.888
Probability of Failure	0.210	0.180	0.183	0.178	0.174
Standard Deviation of Factor of Safety	0.284	0.313	0.303	0.295	0.307

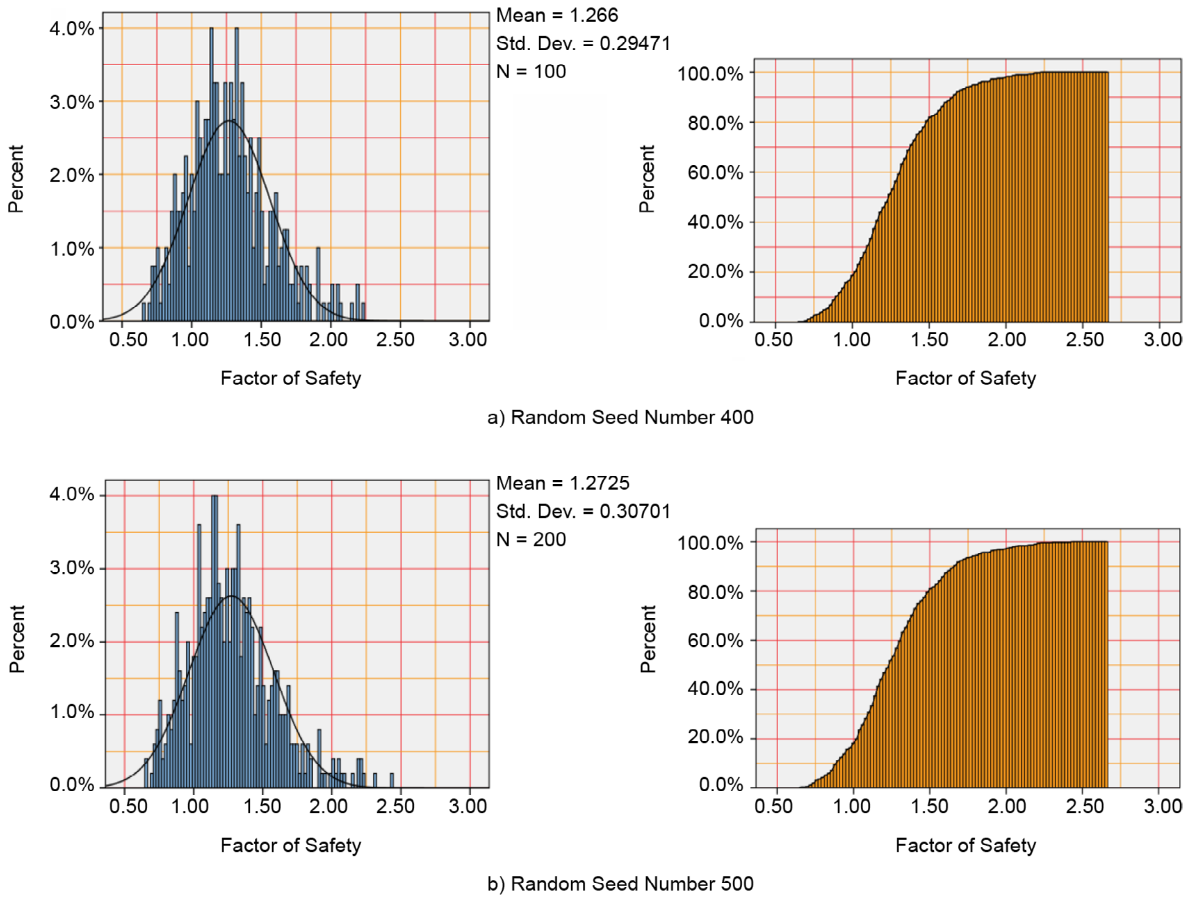

Figure 11. The PDF and CDF of factor of safety for Steady state and pseudo-static condition for random seed number 400 and 500.

Table 8. Compression between the global β and Pf with the common approach of reliability analysis for Steady state and pseudo-static condition.

Seed Number	Reliability Index		Probability of Failure (%)	
	Common Approach	Global β	Common Approach	Global P_f (%)
100	1.175	0.92	9.09	21
200	1.117	0.91	13.568	18
300	1.107	0.88	14.047	18.33
400	1.084	0.91	14.536	17.75
500	1.089	0.89	14.86	17.4

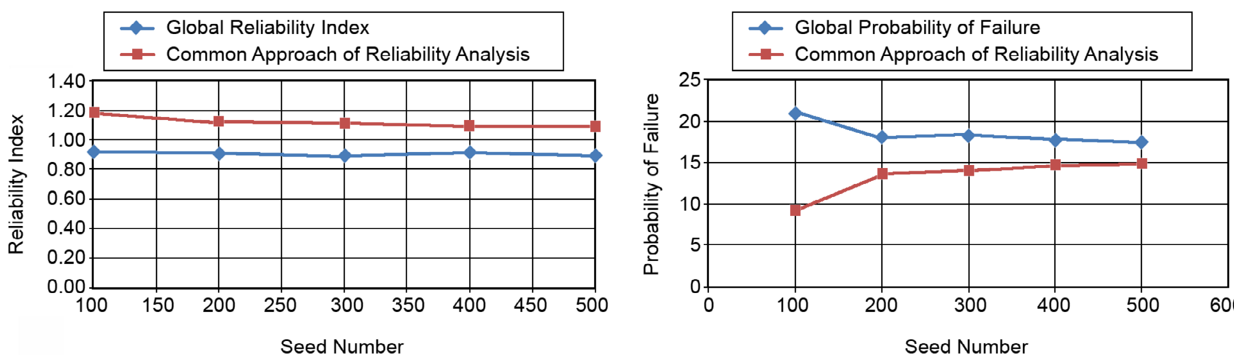

Figure 12. Compression between the global β and P_f with the common approach of reliability analysis for Steady state and pseudo-static condition.

9. Conclusion

In this paper, the common reliability analysis of slope stability problems was studied. In the common way of slope probability evaluation, the focus was on determining the reliability index of critical failure surface. However, there might be some problems with this viewpoint. The minimum reliability index of the slope may not be the same as the reliability index calculated from the common way of reliability analysis. In this paper, the problem was probed by introducing minimum reliability index and the global reliability index, and the results were compared with the common way of reliability analysis.

According to the results of reliability analyses by Monte Carlo simulation technique, the following conclusion may be drawn:

- Considering the reliability of the critical slip surface is not a relevant criterion for evaluating the reliability of the dam.
- The critical slip surface in the deterministic analysis, which has the minimum factor of safety, may not be the same as the surface with the lowest reliability index or the highest probability of failure.
- The common approach of reliability analysis focuses on one slip surface (failure surface). But considering different slip surfaces in the calculation process and estimating the global reliability index may seem more appropriate. The results of this paper demonstrate that the difference between the reliability indexes of these two approaches may be considerable.

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