Seismic Response of Suspension Bridges Under Multi-Component Non-Stationary Random Ground Motion

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ABSTRACT: *A Markov method of analysis is presented for obtaining the seismic response of suspension bridges to nonstationary random ground motion. A uniformly modulated nonstationary model of the random ground motion is assumed which is specified by the evolutionary r.m.s ground acceleration. Both vertical and horizontal components of the ground motion are considered to act simultaneously at the bridge supports. The analysis duly takes into account the angle of incidence of earthquake, the spatial correlation of ground motion and the quasi-static excitation. A suspension bridge is analysed under a set of parametric variations in order to study the nonstationary response of the bridge, The results of the numerical study indicate that (i) frequency domain spectral analysis with peak r.m.s acceleration as input could provide more r.m.s response than the peak r.m.s response obtained by the nonstationary analysis; (ii) longitudinal component of the ground motion significantly influences the vertical vibration of the bridge; and (iii) the angle of incidence of earthquake has considerable influence on the deck response.*

Keywords: Markov method of analysis; Seismic response of suspension bridge; Nonstationary seismic excitation; Modulating function; Quasi-static bridge response

1. Introduction

Recently many studies have been reported on the seismic analysis of suspension bridges [1]. In most cases, the analysis has been performed either for a specified earthquake record or for earthquake assumed to be a stationary random process. Since the non-stationary model of the earthquake process is considered to be more realistic representation of the ground motion, it is important to consider the non-stationary characteristics of the ground motion in the seismic analysis of structures. Seismic analysis of suspension bridges by considering the non-statioanrity of the ground motion is not widely reported in the literature. Hyun et al [8] developed a method for non-stationary analysis of suspension bridges subjected to multi-support excitations which was found to be mainly dependent upon the enveloping function of the time history of ground motion. The non-stationary responses were obtained in terms of time-dependent variance functions. There have been many studies on simpler structures to represent the non-stationary behaviour of these structures under seismic excitations. Lin [10] treated the non-stationary excitation as a sequence of random pulses. By modelling the earthquake as filtered Poisson process, Shinozuka et al [14, 15] developed a procedure to obtain the time-dependent variance of the response. Debchaudhary and Gaspirini [4, 5, 6] developed a method for obtaining the response of multi-degree of freedom systems to non-stationary seismic excitation using Markov approach. The advantage of the Markov approach for the nonstationary analysis of structures for seismic excitation is that it directly obtains the evolutionary *r.m.s* response of the system. Further, the approach does not require the derivation of the evolutionary frequency response function which may be difficult to obtain in many complex structures like, suspension bridges. However, the application of the Markov approach for the response analysis of suspension bridges to non-stationary seismic excitation is not straightforward. It involves some degree of complexity because of (i) horizontal component of ground motion contributing to the vertical vibration of the deck and (ii) the presence of pseudo static component of vertical motion to the total vertical vibration of the bridge deck.

Herein, the vertical response of the bridge deck of suspension bridges to multi-component partially correlated non-stationary random ground motion is obtained using a Markov formulation. Uniformly modulated non-stationary model of the random ground motion is considered in the study. The formulation takes into account the effects of the angle of incidence of earthquake, the ratio between the horizontal and vertical components of ground motion and the quasi-static component of the response. Using the proposed method of analysis, the nonstationary response of a suspension bridge is obtained for a number of parametric variations.

2. Theory

2.1. Seismic Excitation

Seismic excitation is assumed to be multi-component uniformly modulated non-stationary random process. The three components of the ground motion are assumed to be defined in the three principal directions of the earthquake and are assumed to be directed along the principal axes of the bridge x, z, y or shifted with an angle α as shown in Figure (1). The evolutionary *r.m.s* acceleration for each component of the ground motion is specified. The spatial correlation between the seismic excitations at two points is given by a correlation function

$$\rho_{ij} = exp\left(-a\, r_{ij}\right) cos\left(2\pi K_o r_{ij}\right) \tag{1}$$

where a and K_o are parameters which depend on the direction of wave propagation as well as wave type, earthquake type and magnitude. The values of a and K_o are taken as 4.769 and 2.756 respectively [11]. r_{ij} is the distance between the stations i and j measured in the direction of wave propagation which is assumed to coincide with the major principal axis of the ground motion (u) as shown in Figure (1).

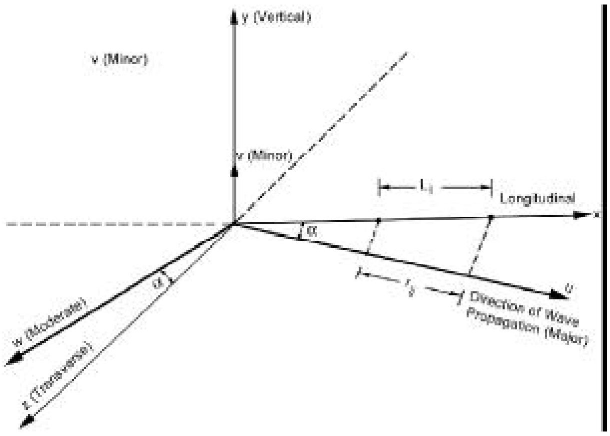

Figure 1. Layout of the principal axes of the bridge (x, y, z) and the principal directions of the ground motion (u, v, w).

The non-stationary support excitations are considered as the outputs of filters excited by the evolutionary white noise. For the formulation of the problem, the filters are augmented at each support degree of freedom and are defined by

$$\begin{aligned} &\ddot{C}_{fi} + 2\xi_{fi}\,\omega_{fi}\,\dot{X}_{fi} + \omega_{fi}^2 X_{fi} = \ddot{S}_i + W_i \\ &\ddot{S}_{fi} + 2\xi_{S_i}\,\omega_{S_i}\,\dot{S}_i + \omega_{S_i}^2 S_i = -W_i \\ &i = 1, 2, \ldots, n_s \end{aligned} \tag{2}$$

where n_s is the number of exciting degrees of freedom i.e. the size of excitation vector. $\{X_f\}$ is the vector of output of filters which is the input to the bridge supports at their degrees of freedom. ω_{fi} and ω_{si} are the i^{th} filter parameters representing the predominant frequencies and the other two parameters ξ_{fi} and ξ_{si} represent the damping ratios. $\{S\}$ is the vector of intermediate response, and $\{W\}$ is the vector of evolutionary white noise having a covariance matrix as

$$\begin{aligned} &\sum\nolimits_{WW}(t, t+\tau) = \\ &\quad E\left\{\left[W(t) - \mu_W(t)\right]\left[W(t+\tau) - \mu_W(t+\tau)\right]^T\right\} \\ &\quad = Q(t).\,\delta(\tau) \end{aligned} \tag{3}$$

where, μ_W is the mean vector of $\{W\}, \delta(\tau)$ is the Dirac delta function and $Q(t)$ is known as the matrix of white noise intensities. By integrating both sides of Eq. (3)

$$\begin{aligned} &\int \sum\nolimits_{ww}(t, t+\tau) = \int Q(t)\,\delta(\tau)\, d\tau \\ &\int \sum\nolimits_{ww}(t, t+\tau)\, d\tau = Q(t) \end{aligned} \tag{4}$$

Thus, $Q(t)$ is the integral of the covariance function of the white noise excitation components. In the present formulation, the elements of the intensity matrix are modelled as piece-wise linear functions of time, although they can take any shape. Typical elements of the covariance matrix are

$$\int\sum_{wiwi}(t,t+\tau)= E\left\{\left[W_i(t)-\overline{W}_i(t)\right]\left[W_i(t+\tau)-\overline{W}_i(t+\tau)\right]^T\right\} = q_{ii}(t).\,\delta(\tau) \quad (5)$$

$$\int\sum_{wiwj}(t,t+\tau)= E\left\{\left[W_i(t)-\overline{W}_i(t)\right]\left[W_j(t+\tau)-\overline{W}_j(t+\tau)\right]^T\right\} = q_{ij}(t).\,\delta(\tau) = \rho_{ij}\left[q_{ii}(t)\,q_{jj}(t)\right]^{1/2}\delta(\tau) \quad (6)$$

ρ_{ij} is the correlation function between excitations corresponding to the i^{th} and j^{th} *d.o.f.* and is given by Eq. (1) and $\overline{W}_i$ and $\overline{W}_j$ are the mean values of the i^{th} and j^{th} elements of the vector $\{W\}$. In general, the ground motion is defined by its free field record which is the output of the filter. The inputs to the filters represent the bed rock excitation and the filters reflect the soil media. It will be subsequently seen that the formulation requires the specification of the matrix $Q(t)$ of the intensities of white noise. The elements of $[Q(t)]$ are determined with the help of the specified evolutionary *r.m.s* acceleration of free field ground motion and the characteristics of the filters.

2.2. Bridge Model and the Equation of Motion

A three-span suspension bridge, as shown in Figure (2), is taken for the formulation of the problem. The bridge is of hinged girder type in each span and the connection between the towers and the cable is of roller type.

The governing equation of motion for the vertical vibration of the i^{th} span of a suspension bridge shown in Figure (2) is given by Pugsley [13]

$$E_i I_i\frac{\partial^4 Y(x_i,t)}{\partial X^4}-H_w\frac{\partial^2 Y(x_i,t)}{\partial X^2}+C_i\frac{\partial Y(x_i,t)}{\partial t} +\frac{\overline{W}_i}{g}\frac{\partial^2 Y(x_i,t)}{\partial t^2}+\frac{\overline{W}_i}{H_w}h(t)=0.0, \qquad i=1,2,3 \quad (7)$$

in which $E_iI_i,\overline{W}_i$ are the flexural rigidity and the dead load of the bridge per unit length of the i^{th} span; H_w is the initial horizontal component of the cable tension due to the dead load; $h(t)$ is the additional horizontal component of the cable tension which includes not only the vibrational part, but also the effect of the vertical movement of all supports, and the longitudinal motion of outer supports and is given by

$$h(t)=\frac{E_c A_c}{L_e}\times\left\{\sum_{i=1}^{3}\left[\frac{\overline{W}_i}{H_w}\int_{i=1}^{3}Y(x_i,t)\,dx_i-\frac{\overline{W}_i L_i}{2H_w}\left(X_{f(i+1)}(t)+X_{fi}(t)\right)\right] +\left(X_{f8}(t)-X_{f5}(t)\right)\right\} \quad (8)$$

in which $X_{fj}(t)$ are the support ground motions as shown in Figure (2).

The solution of Eq. (7) which satisfies the boundary conditions of the stiffening girder of hinged type, gives the expression for the total vertical

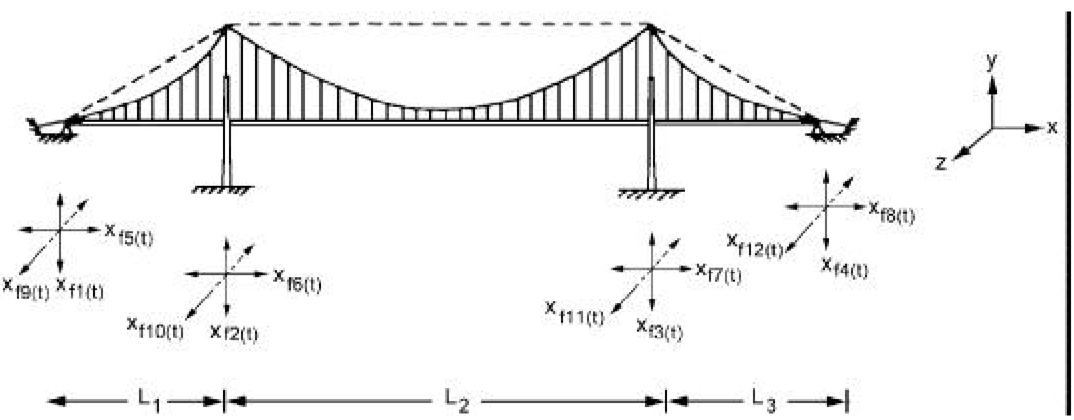

Figure 2. Layout of the suspension bridge under multi-component ground motion.

displacement of the bridge as

$$Y(x_i,t)=y(x_i,t)+\sum_{j=1}^{4}g_{ji}(x_i)X_{fj}(t)\quad ,\ i=\ ,2,3 \tag{9}$$

where $Y(x_i,t), y(x_i,t)$ are the total and the relative vertical displacements of the i^{th} span of the bridge respectively, $g_{ji}(x_i)$ is the quasi-static function governing the vertical displacement in the i^{th} span due to unit vertical displacement of the j^{th} support; $X_{fj}(t), j=1,2,3,4$ are the vertical displacements of the j^{th} support due to the ground motion.

Substituting $Y(x_i,t)$ from Eq. (9) into Eq. (7), and separating the differential equation of quasi-static functions leads to the equation of motion in terms of the relative (dynamic) vertical displacement of the bridge as

$$E_iI_i\frac{\partial^4 y(x_i,t)}{\partial x^4}-H_w\frac{\partial^2 y(x_i,t)}{\partial x^2}+C_i\frac{\partial y(x_i,t)}{\partial t}+\frac{\overline{W_i}}{g}\times\frac{\partial^2 Y(x_i,t)}{\partial t^2}+\frac{\overline{W_i}}{H_w}\left[\frac{E_cA_c}{L_e}\sum_{m=1}^{3}\frac{\overline{W_m}}{H_w}\int_0^{L_m}y(x_m,t)\,dx_m\right]=P(t) \tag{10}$$

$i=1,2,3$

where, E_c, A_c are the modulus of elasticity of the cable material and the cross sectional area of the cable respectively; L_e is the virtual cable length defined as

$$L_e=\int\left(\frac{dS}{dx}\right)^3 dx \tag{11}$$

in which the integration is performed from the hold down to hold down points of the cables and *P*(*t*) is the total force including elastic, damping and inertia forces generated due to the support motions. The quasi-static functions can be obtained by solving the differential equation of the quasi-static functions separated in a similar way as mentioned above. Mode shapes and frequencies for the bridge can be obtained using the continuum approach as given by Chatterjee et al [3].

2.3. State Space Formulation Using Modal Coordinate

The relative vertical displacement $y(x_i,t)$ at any point in the i^{th} span of the bridge deck is given by

$$y(x_i,t)=\sum_{n=1}^{\infty}\Psi_n(x_i)\eta_n(t)\qquad i=1,2,....,N \tag{12}$$

where N is the number of spans, $\Psi_n(x_i)$ is the n^{th} vertical mode shape of the i^{th} span of the bridge and $\eta_n(t)$ is the n^{th} generalized coordinate. Further, the generalized equation of motion for the relative vertical vibration of the bridge deck can be written

$$\ddot{\eta}_n(t)+2\xi_n\omega_n\dot{\eta}_n(t)+\omega_n^2\eta_n(t)=\sum_{j=1}^{12}\alpha_{jn}\ddot{X}_{fj}(t)+\sum_{j=1}^{12}\beta_{jn}\dot{X}_{fj}(t)+\sum_{j=1}^{12}\gamma_{jn}X_{fj}(t) \tag{13}$$

where, $X_{fj}(t)$ is the support displacement corresponding to the j^{th} degree of freedom (Figure (2)) which is the output of a set of filters excited by the evolutionary white noise as given by Eq. (2); $\dot{X}_{fj}(t)$ and $\ddot{X}_{fj}(t)$ are the derivatives of $X_{fj}(t)$ and α_{jn}, β_{jn}, γ_{jn} are the modal participation factors defined as

$$\alpha_{1n}=A_{3n};\alpha_{2n}=A_{6n};\alpha_{3n}=A_{9n};\alpha_{4n}=A_{12n}$$

$$\beta_{1n}=A_{2n};\beta_{2n}=A_{5n};\beta_{3n}=A_{8n};\beta_{4n}=A_{11n}$$

$$\alpha_{jn}=\beta_{jn}=0.0, j=5,6,.....,12$$

$$\gamma_{1n}=A_{1n};\gamma_{2n}=A_{4n};\gamma_{3n}=A_{7n};\gamma_{4n}=A_{10n}$$

$$\gamma_{5n}=A_{13n};\gamma_{8n}=A_{14n}$$

$$\gamma_{jn}=0.0,\quad j=6,7,9,10,11,12$$

where, $A_{1n}, A_{2n},.. .., A_{14n}$ are given in Appendix (I).

Let

$$\left.\begin{aligned}Z_1^j(t)&=S_j(t)\\Z_2^j(t)&=\dot{S}_j(t)\\Z_3^j(t)&=X_{fj}(t)\\Z_4^j(t)&=\dot{X}_{fj}(t)\end{aligned}\right|\quad j=1,2,\quad 12$$

$$\left.\begin{aligned}Z_5^n(t)&=\eta_n(t)\\Z_6^n(t)&=\dot{\eta}_n(t)\end{aligned}\right\}\quad n=1,2,\quad M$$

M is the number of modes included in the analysis.

Then

$$\dot{Z}_1^j(t)=\dot{S}_j(t)=Z_2^j(t)$$

$$\dot{Z}_2^j(t)=\ddot{S}_j(t)=-\omega_{sj}^2S_j(t)-2\xi_{sj}\omega_{sj}(t)\dot{S}_j(t)-W_j(t)$$
$$=-\omega_{sj}^2Z_1^j(t)-2\xi_{sj}\omega_{sj}Z_2^j(t)-W_j(t)$$

$$\dot{Z}_3^j(t)=\dot{X}_{fj}(t)=Z_4^j(t)$$

$$\dot{Z}_4^j(t) = \ddot{X}_{fj}(t) = \tag{19}$$
$$-\omega_{sj}^2 S_j(t) - 2\xi_{sj}\omega_{sj}\dot{S}_j(t) - \omega_{fj}^2 X_{fj}(t) - 2\xi_{fj}\omega_{fj}\dot{X}_{fj}(t)$$
$$= -\omega_{sj}^2 Z_1^j(t) - 2\xi_{sj}\omega_{sj} Z_2^j(t) - \omega_{fj}^2 Z_3^j(t) - 2\xi_{fj}\omega_{fj} Z_4^j(t)$$

where $j = 1, 2, \ldots, 12$ and $W_j(t)$ can be defined as

$$W_j(t) = W_u cos\alpha - W_w sin\alpha \quad j = 5,6,7,8\,(x\text{-}direction)$$

$$W_j(t) = W_v \quad j = 1,2,3,4\,(y\text{-}direction)$$

$$W_j(t) = W_u sin\alpha + W_w cos\alpha \quad j = 9,10,11,12\,(z\text{-}direction)$$

where W_u, W_v, W_w are the three components of the ground motion and α is the angle between the major component (u) and the longitudinal direction (x) of the bridge, see Figure (2).

$$\left.\begin{aligned}\dot{Z}_5^n(t) &= \dot{\eta}_n(t) = Z_6^n(t)\\ \dot{Z}_6^n(t) &= \ddot{\eta}_n(t)\end{aligned}\right\} n = 1,2,\ldots M \tag{20}$$

Now Eq. (13) can be written as

$$\ddot{\eta}_n(t) =$$
$$-\omega_n^2\eta_n(t) - 2\xi_n\omega_n\dot{\eta}_n(t)$$
$$+\sum_{j=1}^{12}\alpha_{jn}\Big[-\omega_{sj}^2 Z_1^j(t) - 2\xi_{sj}\omega_{sj} Z_2^j(t)$$
$$-\omega_{fj}^2 Z_3^j(t) - 2\xi_{fJ}\omega_{fj} Z_4^j(t)\Big]$$
$$+\sum_{j=1}^{12}\beta_{jn} Z_4^j(t) + \sum_{j=1}^{12}\gamma_{jn} Z_3^j(t)$$

$$= -\omega_n^2 Z_5^n(t) - 2\xi_n\omega_n Z_6^n(t)$$
$$+\sum_{j=1}^{12}\Big[-\alpha_{jn}\,\omega_{sj}^2 Z_1^j(t) - 2\alpha_{jn}\xi_{sj}\omega_{sj} Z_2^j(t)$$
$$-\alpha_{jn}\omega_{fj}^2 Z_3^j(t) - 2\alpha_{jn}\xi_{fJ}\omega_{fj} Z_4^j(t)\Big] \tag{21}$$
$$+\sum_{j=1}^{12}\beta_{jn} Z_4^j(t) + \sum_{j=1}^{12}\gamma_{jn} Z_3^j(t)$$

Equation of motion for the bridge in terms of state variables can be written in the matrix form as

$$\{\dot{Z}(t)\} = [A]\{Z(t)\} + [B]\{W(t)\} \tag{22}$$

where $\{Z(t)\}$ is the vector of state variables given by

$$[\{Z(t)\}]^T =$$
$$\left[Z_1^1 Z_2^1 Z_3^1 Z_4^1 \quad Z_1^{12} Z_2^{12} Z_3^{12} Z_4^{12} : Z_5^1 Z_6^2 . \quad Z_5^M Z_6^M\right] \tag{23}$$

The matrices $[A]$ and $[B]$ are given in the Appendix (II).

$\{W(t)\}$ is the vector of the white noise arranged

$$[\{W(t)\}]^T = [W_1(t). \quad W_j(t). \quad W_{12}(t)] \tag{24}$$

Eq. (22) describes a system of ($4 \times 12 + 2 \times M$) first order differential equations. The solution of Eq. (22) in time domain is given by

$$Z(t)\} = e^{[A](t-t_0)}\{Z(t_0)\} + \int_{t_0}^{t} e^{[A](t-\tau)}[B]\{W(\tau)\}d\tau \tag{25}$$

The Matrix exponential in Eq. (25) is defined by the power series as

$$e^{[A]t} = \sum_{k=0}^{\infty}\frac{t^k}{k!}[A]^k \tag{26}$$

Eq. (25) can be written as

$$Z(t)\} = [\phi(t,t_0)]\{Z(t_0)\} + \int_{t_0}^{t}[\phi(t,\tau)][B]\{W(\tau)\}d\tau \tag{27}$$

where

$$[\phi(t,t_0)] = e^{[A](t-t_0)} \tag{28}$$

$[\phi(t,t_0)]$ is the state transition matrix and may be calculated in different ways. Here, the method of similarity transformation of matrix A is used [12, 16].

2.4. Evolutionary Mean and Covariance Matrix of State Vector {Z}

If the excitation $\{W(t)\}$ is evolutionary Gaussian white noise, the response $\{Z(t)\}$ is an evolutionary Gauss-Markov random process [9]. The mean vector of $\{w(t)\}$ is

$$E[\{W(t)\}] = \{\mu_w(t)\} \tag{29}$$

and the auto covariance matrix of $\{W(t)\}$ is

$$\begin{aligned}\Sigma_{ww}(t_1,t_2) &= E\big[\{W(t_1) - \mu_w(t_1)\}\{W(t_2) - \mu_w(t_2)\}^T\big]\\ &= Q_w(t_1)\delta(t_1 - t_2)\end{aligned} \tag{30}$$

where $Q_w(t_1)$ is the matrix of intensities of white noise vector $\{W(t)\}$ as defined earlier; $\delta(t_1 - t_2)$ is Dirac delta function.

The governing equation for the evolutionary mean vector $\{Z(t)\}$ is

$$\{\dot{\mu}_z(t)\}=[A]\{\mu_z(t)\}+[B]\{\mu_w(t)\} \quad (31)$$

The solution of this equation in terms of mean, assuming that the response vector at time (t_0) being $\{Z(t_0)\}$, is

$$\{\mu_z(t)\}=[\phi(t,t_0)]\{\mu_z(t_0)\}+\int_{t_0}^{t}[\phi(t,\tau)][B]\{\mu_w(\tau)\}d\tau \quad (32)$$

The covariance matrix of the state vector $Z(t)$ can be calculated as

$$\sum_{zz}(t_1,t_2)=E\left[\left\{\underline{Z}(t_1)-\underline{\mu}_z(t_1)\right\}\left\{\underline{Z}(t_2)-\underline{\mu}_z(t_2)\right\}^T\right] \quad (33)$$

Substituting Eq. (32) into Eq. (33), the following expression for the covariance matrix is obtained

$$\sum_{zz}(t_1,t_2)=\underline{\phi}(t_1,t_0)\sum_{zz}(t_0)\underline{\phi}(t_2,t_0)^T$$
$$+\int_{t_0}^{t_1}\int_{t_0}^{t_2}\underline{\phi}(t_1,\tau_1)\underline{B}\,\underline{Q}(\tau_1)\delta(\tau_1-\tau_2) \quad (34)$$
$$\times\underline{B}^T\underline{\phi}^T(t_2,\tau_2)d\tau_1 d\tau_2$$

The integral term is obtained by assuming $t_1>t_2, t_0\le\tau_1\le t_1$ and $t_0\le\tau_2\le t_2$ and the integration is first performed with respect to τ_1, then with respect to τ_2. In this case,

$$\sum_{zz}(t_1,t_2)=\underline{\phi}(t_1,t_0)\sum_{zz}(t_0)\underline{\phi}^T(t_2,t_0)$$
$$+\int_{t_0}^{t_2}\underline{\phi}(t_1,\tau_2)\underline{B}\,\underline{Q}(\tau_2)\underline{B}^T\underline{\phi}^T(t_2,\tau_2)d\tau_2 \quad (35)$$

putting $t_1=t_2=t$, the covariance matrix is

$$\underline{\sum_{zz}}(t)=\underline{\phi}(t,t_0)\underline{\sum_{zz}}(t_0)\underline{\phi}^T(t,t_0)$$
$$+\int_{t_0}^{t}\underline{\phi}(t,\tau)\underline{B}\,\underline{Q}(\tau)\underline{B}^T\underline{\phi}^T(t,\tau)d\tau \quad (36)$$

If the mean of the exciting vector is assumed to be zero (i.e. $\underline{\mu}_w=0$), then Eq. (36) fully describes the state output vector $\{Z(t)\}$. Thus, the covariance matrix of response can be calculated at any time t provided that the covariance matrix at any previous time t_0, and the matrix of strengths of the excitation (i.e. intensity matrix $[Q(t)]$ of $\{W(t)\}$) are known.

2.5. Calculation of Intensity Matrix Q(t) for the Input White Noise, Given the r.m.s Ground Acceleration

The fictitious piece-wise linear-strength envelope (intensity function of white noise) needed to match any desired ground motion can be directly obtained by analyzing the filters before augmenting them to the bridge system.

Consider any filter corresponding to any excitation *d.o.f.* (*j*), referring to Eq. (2)

$$\ddot{X}_{fj}+2\xi_{fj}\omega_{fj}\dot{X}_{fj}+\omega_{fj}^2X_{fj}=\ddot{S}_j+W_j$$
$$\ddot{S}_{fj}+2\xi_{Sj}\omega_{Sj}\dot{S}_j+\omega_{Sj}^2S_{fj}=-W_j \quad (37)$$

Where j = 5, 6, 7, 8 correspond to x-direction excitations, j = 1, 2, 3, 4 correspond to y-direction excitations and j = 9, 10, 11, 12 correspond to z-direction excitations as shown in Figure (2).

The filter equations j^{th} degree of freedom may be written in the matrix form as

$$\{\dot{Z}(t)\}^j=[A]^j\{Z(t)\}^j+[B]^j\{W(t)\}^j \quad (38)$$

where

$$\left[\{Z(t)\}^j\right]^T=[Z_1\,Z_2\,Z_3\,Z_4]^j \quad (39)$$

$\{\overline{W}(t)\}^j$ is the vector of white noise for the filter. The matrices $[A]^j$ and $[B]^j$ are given in the Appendix (II).

The solution of Eq. (38) for filters is similar to that given by Eq. (22). Assuming $\{\overline{W}(t)\}$ to be zero mean random vectors, the covariance matrix of the filter responses is given by

$$\left[\sum_{Z_iZ_j}(t)\right]=[\phi_i(t,t_0)]\left[\sum_{Z_iZ_j}(t_0)\right][\phi_j(t,t_0)]^T$$
$$+\int_{t_0}^{t}[\phi_i(t,\tau)][B]_i[Q(\tau)][B]_j^T[\phi_j(t,\tau)]^Td\tau \quad (40)$$

where $\phi_i(t,t_0)$ is the transition matrix for filter i. For the case in which the elements of $[Q(t)]$ are piece-wise linear, as shown in Figure (3), Eq. (40) can be written in the following form: (at time $t_1=Dt$ i.e. point 1 with $t_0=0.0$). Note that the duration is divided into increments Dt, and q is a single element for each filter which is one element of matrix $[Q(t)]$

$$\left[\sum_{Z_iZ_j}(Dt)\right]_1=[\phi_i(Dt)]\left[\sum_{Z_iZ_j}(0)\right][\phi_j(Dt)]^T$$
$$+q_0\int_0^{Dt}[\phi_i(Dt,\tau)][B]_i[B]_j^T[\phi_j(Dt-\tau)]^Td\tau \quad (41)$$
$$+\frac{q_1-q_0}{Dt}\int_0^{Dt}[\phi_i(Dt-\tau)][B]_i[B]_j^T[\phi_j(Dt-\tau)]^T\tau\,d\tau$$

For point 2, i.e. $t_2=2Dtq=q_2$,

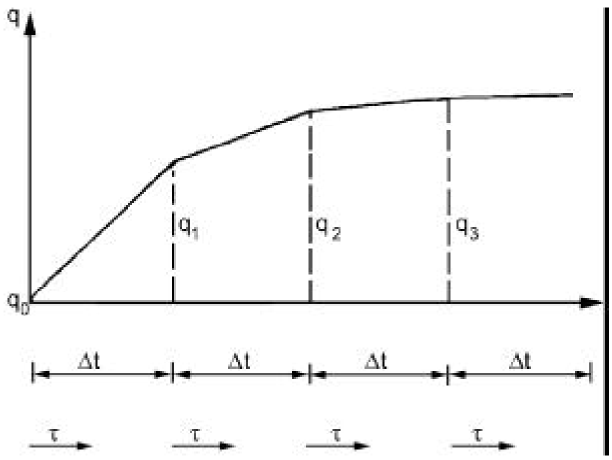

Figure 3. Piece-wise linear strength intensity function.

$$\left\lfloor \Sigma_{Z_iZ_j}(t_2) \right\rfloor_1 = [f_i(t_2-t_1)]\left[\Sigma_{Z_iZ_j}(t_1)\right][f_j(t_2-t_1)]^T + q_1[F_{10}] + \frac{q_2-q_1}{Dt}[F_{20}] \quad (42)$$

where

$$[F_{10}] = \int_0^{Dt} [\phi_i(Dt-\tau)][B]_i[B]_j^T[\phi_j(Dt-\tau)]^T d\tau$$

$$[F_{20}] = \int_0^{Dt} [\phi_i(Dt-\tau)][B]_i[B]_j^T[\phi(Dt-\tau)]^T \tau d\tau \quad (43)$$

$[F_{10}]$, $[F_{20}]$ are 4×4 matrices. They are constant for equal intervals Dt.

From Eqs. (41) and (42), a recursive relation for covariance and cross covariance of state vector can be obtained as

$$\left[\Sigma_{ij}(t_k)\right] = [f_i(Dt)]\left[\Sigma_{ij}(t_{k-1})\right][f_j(Dt)]^T + q_{k-1}[F_{10}] + \frac{q_k - q_{k-1}}{Dt}[F_{20}] \quad (44)$$

Generally, the ground motion is represented in terms of *r.m.s* ground acceleration, which can be written as

$$\ddot{X}_f = \left[-\omega_S^2 - 2\xi_S\omega_S - \omega_f^2 - 2\xi_f\omega_f\right]\{Z\} = [d]\{Z\} \quad (45)$$

Since Eq. (45) is linear

$$\sigma_{\ddot{X}_f}^2 = [d][\Sigma_{ZZ}][d]^T \quad (46)$$

Using Eqs. (44) and (46), Cross variance of ground acceleration at time t_k is given by

$$\begin{aligned}\sigma_{ij}^2(t_k) = {} & \underline{d}\,\underline{\phi}_i(Dt)\Sigma_{Z_iZ_j}(t_{k-1})\underline{\phi}_j^T(Dt)\underline{d}^T \\ & + q_{ij}(t_{k-1})\underline{d}[F_{10}]\underline{d}^T + \frac{q_{ij}(t_k)-q_{ij}(t_{k-1})}{Dt}\underline{d}[F_{20}]\underline{d}^T \\ = {} & \underline{d}\,\underline{\phi}_i(Dt)\Sigma_{Z_iZ_j}(t_{k-1})\underline{\phi}_j^T(Dt)\underline{d}^T \\ & + q_{ij}(t_{k-1}).A_0 + \frac{q_{ij}(t_k)-q_{ij}(t_{k-1})}{Dt}.B_0\end{aligned} \quad (47)$$

$$\sigma_{ij}^2(t_k) = \underline{d}\,\underline{\phi}_i(Dt)\Sigma_{Z_iZ_j}(t_{k-1})\underline{\phi}_j^T(Dt)\underline{d}^T + q_{ij}(t_{k-1}).\left(A_0 - \frac{B_0}{Dt}\right) + \frac{q_{ij}(t_k)}{Dt}B_0 \quad (48)$$

where $A_0 = \underline{d}[F_{10}]\underline{d}^T$ and $B_0 = \underline{d}[F_{20}]\underline{d}^T$

It is known that $\sigma_{ij}^2(t_k)$ can be written as

$$\sigma_{ij}^2(t_k) = \sigma_i(t_k)\sigma_j(t_k)\rho_{ij} \quad (49)$$

where ρ_{ij} is the correlation function between the two filters i and j.

Using Eq. (49) and Eq. (48) can be written as

$$q_{ij}(t_k) = \frac{Dt}{B_0}\Big[s_i(t_k)s_j(t_k)r_{ij} - \underline{d}\,\underline{f}_i(Dt) \times \Sigma_{Z_iZ_j}(t_{k-1})\underline{f}_j^T(Dt)\underline{d}^T - F_0 q_{ij}(t_{k-1})\Big] \quad (50)$$

where

$$F_0 = \left(A_0 - \frac{B_0}{Dt}\right)$$

and

$$\Sigma_{Z_iZ_j}(t_k) = \underline{\phi}_i(Dt)\Sigma_{Z_iZ_j}(t_{k-1})\underline{\phi}_j^T(Dt) + q_{ij}(t_{k-1})[F_{10}] + \frac{q_{ij}(t_k)-q_{ij}(t_{k-1})}{Dt}.[F_{20}] \quad (51)$$

Now, defining the *r.m.s* time history of ground acceleration and filter characteristics, the corresponding time history of elements $q_{ij}(t)$ of the matrix $[Q(t)]$ can be easily calculated by using Eqs. (50) and (51). The matrix $[Q(t)]$ at each time "t" is to be used in Eq. (36) to calculate the covariance matrix for the bridge response.

2.6. Calculation of the Bridge Responses

Once the evolutionary mean and the covariance matrices for the total state vector are computed, the evolutionary mean and covariance matrix of any desired response can be obtained by expressing them as a linear function of the state variables that includes the base displacement, modal coordinates, and their derivatives.

Let $\{U(t)\}$ be any unknown response vector that is of interest and is linearly related to some other response vector $\{R(t)\}$, whose evolutionary mean vector $\{\overline{R}(t)\}$ and covariance matrix $\sum_{RR}(t)$ are already known, i.e.,

$$\{U(t)\}=[D]\{R(t)\} \tag{52}$$

Then, the evolutionary mean and the covariance matrices of the responses $\{U(t)\}$ can be expressed in terms of the known evolutionary mean and covariance matrices of the response $\{R(t)\}$ as follows:

$$\{\overline{U}(t)\}=[D]\{\overline{R}(t)\} \tag{53}$$

$$\sum_{UU}(t)=[D]\sum_{RR}[D]^T \tag{54}$$

Since the input is assumed to be zero mean random vector, the output will be zero mean, and response vector will be zero mean and the diagonal terms will give the evolutionary mean square value of the response.

2.7. Evolutionary Mean Square Responses for Suspension Bridge

2.7.1. Evolutionary Mean Square Displacement

The expression for the total displacement at any time in the i^{th} span of the suspension bridge can be written as

$$Y(x_{i,t})=[\psi(x_i)]\{\eta\}+[G(x_i)]\{X_f\} \tag{55}$$

where

$$[\psi(x_i)]=[\psi_1(x_i)\ldots.\ \psi_M(x_i)]$$

$$[\{\eta\}]^T=[\eta_1\ \ldots\ldots..\ \eta_M]$$

$$[G(X_i)]=[g_1(x_i)\ g_2(x_i)\ g_3(x_i)\ g_4(x_i)]$$

$$[\{X_f\}]^T=[X_{f1}\ X_{f2}\ X_{f3}\ X_{f4}]$$

Then, the evolutionary mean square of the total displacement can be written as

$$\begin{aligned}\sigma_Y^2(X_i,t)=&[\psi(x_i)][\textstyle\sum_{\eta\eta}(t)][\psi(x_i)]^T\\&+[G(x_i)][\textstyle\sum_{X_fX_f}(t)][G(x_i)]^T\\&+[\psi(x_i)][\textstyle\sum_{\eta X_f}(t)][G(x_i)]^T\\&+[G(x_i)][\textstyle\sum_{X_f\eta}(t)][\psi(x_i)]^T\end{aligned} \tag{56}$$

where $[\sum_{\eta\eta}(t)][\sum_{X_fX_f}(t)][\sum_{\eta X_f}(t)]$ and $[\sum_{X_f\eta}(t)]$ can be easily assembled from the covariance matrix of the state variable $[\sum_{zz}(t)]$.

2.7.2. Evolutionary Mean Square Bending Moment and Cable Tension h(t)

The evolutionary mean square value of the bending moment can be easily obtained by using $EI\dfrac{d^2\psi}{dx^2}$ and $EI\dfrac{d^2G}{dx^2}$ instead of $\psi(x_i)$ and $G(x_i)$ respectively. Further, the evolutionary mean square value of the additional horizontal component of cable tension $h(t)$ can also be obtained.

3. Numerical Study

Three span suspension bridge [7] shown in Figure (2) is considered for the parametric study. The following data are used in the study.

$$L_1=L_3=155.0m;\ L_2=460.0m;$$

$$\overline{W}_1=\overline{W}_2=\overline{W}_3=5347kg/m$$

$$I_1=I_3=0.3749m^4;\ I_2=0.3269m^4;\ A_c=0.078m^2$$

$$H_w=30038.0\times10^3N;\ L_{e1}=L_{e3}=281.0m;\ L_{e2}=494m;$$

$$E_c=1.86\times10^{11}N/m^2;\ E=1.998\times10^{11}N/m^2.$$

The stiffening girder in each span is hinged at the ends and the cable is free to move at the tower top (i.e. roller type cable connection). Uniformly modulated non-stationary ground motion is expressed in terms of the evolutionary *r.m.s* ground acceleration. Three modulating functions, shown in Figure (4), are considered in the study. The value of $\sigma_{\ddot{f}_g}$ (the peak value of the *r.m.s* ground acceleration) is

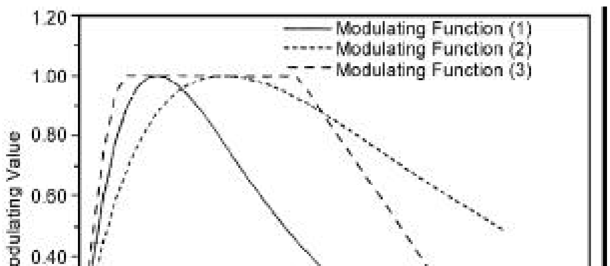

Figure 4. Modulating functions used in the parametric study.

taken as 0.61 $^{m/s2}$. The ground motion is described along the principal directions of the earthquake by specifying different proportions between R_u, R_v, R_w. For the angle of incidence $\alpha = 0.0°$, the ground motions in the three directions refer to those corresponding to the longitudinal, vertical and transverse directions of the bridge. Three sets of filter parameters are used representing soft, firm and very firm soils respectively and are shown in Table (1). The strength intensity functions for the set of filter parameters describing the firm soil for the three modulating functions are shown in Figure (5). It is seen that the maximum value of the strength intensity function occurs at the same time where the corresponding modulating function attains its peak. However, the shapes of the strength intensity functions are not exactly the same as those of the corresponding modulating functions. The evolutionary *r.m.s* responses are calculated with $R_u : R_v : R_w = 1.0:1.0:1.0; \alpha = 0.0°$ and the set of filter parameters corresponding to the firm soil condition, unless mentioned otherwise. With the help of the numerical study, effects of different important parameters on the responses of the bridge are investigated.

Table 1. Filter parameters corresponding to different soil conditions.

Soil Condition	Filter Parameters			
	ω_s	ω_f	ξ_s	ξ_f
Soft Soil	6.2832	0.62832	0.40	0.40
Firm Soil	15.708	1.5708	0.60	0.60
Very Firm Soil	31.416	3.1416	0.80	0.80

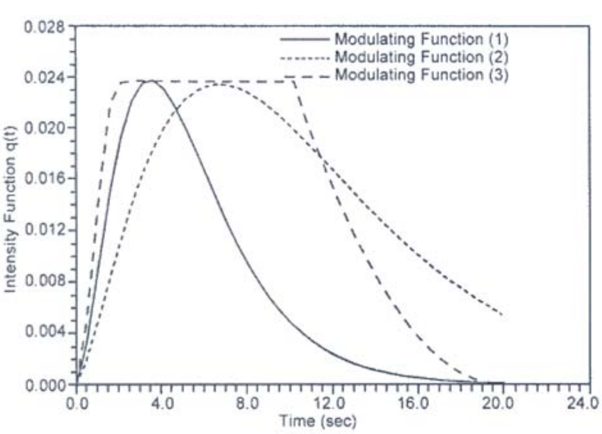

Figure 5. Strength intensity functions correspnding to the modulating functions.

3.1. Effect of the Degree of Nonstationarity (The Nature of Modulating Function)

The degree of nonstationarity is denoted by the sharpness of the modulating function with time. The sharper the variation of the modulating function with time, more is the degree of nonstationarity.

Figures (6) to (8) show the evolutionary *r.m.s*

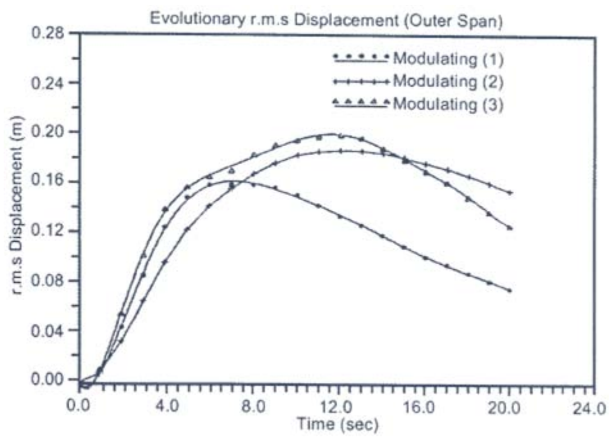

Figure 6. Evolutionary *r.m.s* of vertical displacement at the mid-point of the outer span.

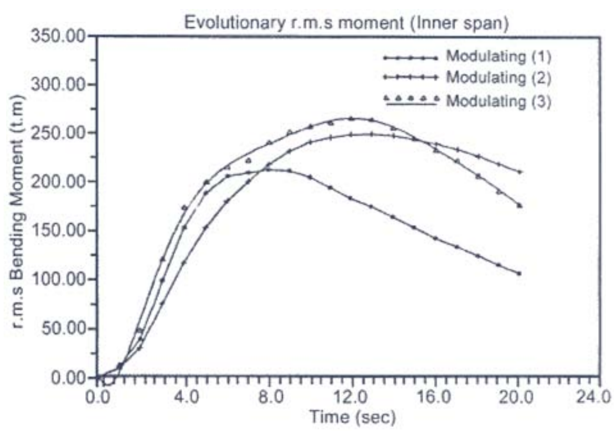

Figure 7. Evolutionary *r.m.s* of vertical bending moment at the mid-point of the inner span.

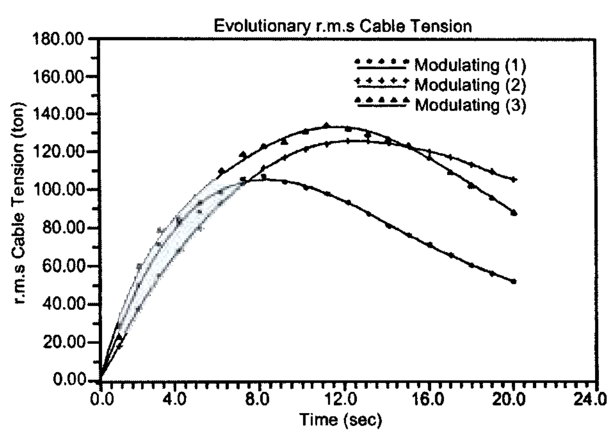

Figure 8. Evolutionary *r.m.s* of additional horizontal component of the cable tension h(t).

responses (standard deviation = *r.m.s* value since the random process is assumed with zero mean) for the displacement at the mid-point of the outer span, the bending moment at the mid-point of the inner span and the additional horizontal component of the cable tension $h(t)$ for the three modulating functions. It is seen that the nature of the evolutionary response and its maximum value depend upon the modulating function being used. The sharper the modulating function (modulating function (1)), less is the maximum value of the *r.m.s* response. Further, the variation of the *r.m.s* response with time is different than that of the corresponding modulating function with time. The effect of nonstationarity on the response is shown by the difference between the maximum *r.m.s* response as obtained from the stationary analysis (frequency domain spectral analysis). The latter is determined with input as Clough and Penzien double filtered *PSDF* of ground acceleration having $\sigma_{\ddot{f}_g} = 0.61 m/s^2$ (i.e. the peak *r.m.s* acceleration in the evolutionary input).

Table (2) compares between the maximum *r.m.s* responses as obtained by the nonstationary analysis and the *r.m.s* responses as obtained by the frequency domain spectral analysis (stationary analysis). The difference between the stationary *r.m.s* responses and the maximum value of the nonstationary *r.m.s* responses is about 17% for the modulating function (3), about 25% for the modulating function (2) and about 45% for the modulating function (1). For further parametric studies, the modulating function (2) is used.

3.2. Effect of the Ratio Between the Three Components of Ground Motion

Table (3) shows the effect of the ratio between the

Table 2. Effect of the nature of modulating function on the *r.m.s* responses.

Position	Stationary	Non-Stationary		
		Modulating (1)	Modulating (2)	Modulating (3)
Outer Span Displacement (m)	0.2253	0.1598	0.1876	0.2002
Outer Span Moment (t.m)	690	492	577	616
Inner Span Displacement (m)	0.3628	0.2491	0.2915	0.3102
Inner Span Moment (t.m)	303	213	250	268
h(t) (ton)	152	107	126	134

Table 3. Effect of the ratio between the three components of ground motion on the *r.m.s* responses (R_u: R_v: R_w).

Position	1.0:0.4:0.6		0.6:0.5:0.6		0.8:0.5:0.6	
	Stationary	Non-Stationary•	Stationary	Non-Stationary•	Stationary	Non-Stationary•
Outer Span Displacement (m)	0.1988	0.1641	0.1296	0.1076	0.1651	0.1366
Outer Span Moment(t.m)	617	511	399	332	511	424
Inner Span Displacement (m)	0.3499	0.2813	0.2149	0.1727	0.2828	0.2273
Inner Span Moment(t.m)	272	224	175	144	225	185
h(t) (ton)	138	114	88	73	114	94

• Peak value of the evolutionary *r.m.s* response (modulating function (2)).

three components of the ground motion on the maximum *r.m.s* responses obtained from the non-stationary analysis and the *r.m.s* responses obtained from the stationary analysis. Three different ratios $(R_u : R_v : R_w)$ between the three components of the ground motion are considered for the study namely, (i) 1.0:0.4:0.6; (ii) 0.6:0.5:0.6; (iii) 0.8:0.5:0.6. The angle of incidence of the earthquake is taken as $\alpha = 0.0°$, i.e. the three components of the ground motion coincide with the principal directions of the bridge (*x*, *y*, *z*). It is seen that the relative magnitude of the *x* component of the ground motion has a predominant effect on the responses at the mid points of the outer and the inner spans. This is expected since the horizontal movement of the abutments (in the longitudinal direction) substantially influences the vertical movement of the bridge deck because of the fluctuation in the cable tension. It is observed that as $R_u(=R_x)$ increases, the response also increases. Even if $R_z(=R_w)$ is decreased but R_x is increased $(=R_u)$, and the vertical response of the bridge deck increases.

3.3. *Effect of the Nature of the Filter Coefficients (Soil Conditions)*

Three different filter coefficients denoting three different soil conditions have been used in the study. The evolutionary strength functions for the three soil conditions are shown in Figure (9). Although the evolutionary free field *r.m.s* ground acceleration is same for all hree soil conditions, the shapes of the evolutionary strength functions and the evolutionary *r.m.s* responses are different for different soil conditions. This is the case because the filter coefficients modify the frequency contents of the free field ground motion, see Eq. (2). Table (4) compares between the maximum *r.m.s* responses obtained from the non-stationary analysis and the *r.m.s* responses as obtained from the stationary analysis. It is seen from the table that the responses are more for the soft soil condition and the difference between the responses obtained by the stationary and non-stationary analyses remains nearly the same for all three soil conditions.

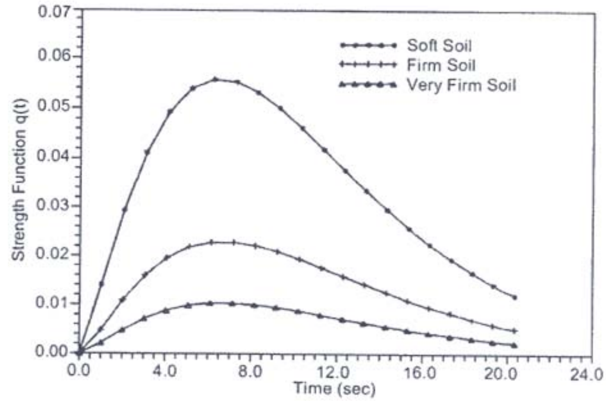

Figure 9. Strength intensity functions for different soil parameters (soil condition).

3.4. *Effect of the Angle of Incidence of Earthquake* (α)

The effect of the angle of incidence (α) on the displacement response is shown in Table (5). $\alpha = 0.0°$ corresponds to the case when the major principal component of the earthquake is along the longitudinal direction of the bridge and $\alpha = 90°$ indicates the case when the moderate principal component of the earthquake is along the longitudinal axis of the bridge, see Figure (1). The minor principal component of the earthquake is always in the vertical direction. Further,

Table 4. Effect of the filter coefficients (soil conditions) on the *r.m.s* responses.

Position	Soft Soil		Firm Soil		Very Firm Soil	
	Stationary	Non-Stationary*	Stationary	Non-Stationary*	Stationary	Non-Stationary*
Outer Span Displacement (m)	0.5772	0.4933	0.2253	0.1876	0.0557	0.0476
Outer Span Moment(t.m)	1571	1309	690	577	177	151
Inner Span Displacement (m)	1.4617	1.3345	0.3628	0.2915	0.0627	0.0500
Inner Span Moment(t.m)	849	739	303	250	79	68
h(t) (ton)	360	300	152	126	43	37

Table 5a. Effect of the angle of incidence of the earthquake (α) on the *r.m.s* responses.

Point	$\alpha = 0.0^o$		$\alpha = 35^o$		$\alpha = 65^o$	
	Stationary	Non-Stationary•	Stationary	Non-Stationary•	Stationary	Non-Stationary•
Outer Span Displacement (m)	0.2253	0.1876	0.1966	0.1631	0.2502	0.2076
Outer Span Moment (t.m)	690	577	606	505	772	643
Inner Span Displacement (m)	0.3628	0.2915	0.3308	0.2660	0.4185	0.3363
Inner Span Moment (t.m)	303	250	280	231	347	286
h(t) (ton)	152	126	133	110	172	142

• Peak value of the evolutionary *r.m.s* response (modulating function (2)).

Table 5b. Effect of the angle of incidence of the earthquake (α) on the *r.m.s* responses.

Point	$\alpha = 75^o$		$\alpha = 80^o$		$\alpha = 90^o$	
	Stationary	Non-Stationary•	Stationary	Non-Stationary•	Stationary	Non-Stationary•
Outer Span Displacement (m)	0.1697	0.1431	0.2661	0.2223	0.1491	0.1322
Outer Span Moment (t.m)	508	429	813	680	426	376
Inner Span Displacement (m)	0.2345	0.1875	0.4129	0.3319	0.1098	0.0880
Inner Span Moment (t.m)	199	165	325	267	187	165
h(t) (ton)	102	85	164	135	106	95

• Peak value of the evolutionary *r.m.s* response (modulating function (2)).

denotes the case of fully correlated excitations. As the ratio between the three components of the earthquake is taken as 1.0:1.0:1.0, the change in predominantly effects the correlation between excitations at any two points by modifying the separation length , see Figure (1).

The table shows that the maximum response at any section of the bridge deck does not necessarily occur for , it may occur for an angle of incidence between 0 to The responses are minimum for i.e., for fully correlated ground motion. The critical value of depends upon the section at which the response is desired. Further, the difference between the maximum *r.m.s* response and the *r.m.s* response as obtained by the stationary and non-stationary analyses varies with The value of for which this difference becomes maximum depends upon the response quantity of interest and the section at which the response is desired.

4. Conclusions

Seismic response of the suspension bridge to multi-component non-stationary partially correlated random ground motion is obtained using a Markov approach. An uniformly modulated non-stationary model of the random ground motion is assumed and is specified by the evolutionary *r.m.s* ground acceleration. The analysis duly takes into account the spatial correlation of the ground motion, angle of incidence of earthquake and the quasi-static excitation. Using the proposed method of analysis, a suspension bridge is analyzed under a set of parametric variations in order to study the non-stationary response behaviour of the bridge. The results of the numerical study show that

- ❖ The shape of the modulation function depicting the degree of nonstationarity significantly influences the evolutionary *r.m.s* response of the bridge. The effect of nonstationarity is to decrease the *r.m.s* response.
- ❖ Frequency domain spectral analysis (stationary analysis) provides higher *r.m.s* responses compared to the maximum *r.m.s* responses obtained by the non-stationary analysis; the difference could be as much as 45%.
- ❖ The sharper the modulating function, more is the difference between the maximum *r.m.s* response (of the non-stationary analysis) and the stationary *r.m.s* response.
- ❖ Responses are more for the filter coefficients corresponding to soft soil condition. However, the difference between the maximum *r.m.s* response (of the non-stationary analysis) and the stationary *r.m.s* response remains nearly the same for all soil conditions.
- ❖ Fully correlated ground motion provides less value of the response.
- ❖ The maximum response does not occur for zero angle of incidence of earthquake (i.e. major component coinciding with the longitudinal axis of the bridge). The critical angle of incidence depends upon the response quantity of interest. Further, the difference between the maximum *r.m.s* response (of the non-stationary analysis) and the stationary *r.m.s* response differs with the angle of incidence of earthquake.
- ❖ Longitudinal component of ground motion significantly influences the vertical vibration of the bridge deck.

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Appendix I

The elements of $[A_n]$ matrix

$$A_{1n} = \mu_n \delta \left(\frac{\overline{W}_1 L_1}{2H_w} \right);$$

$$A_{2n} = -2\xi_n \omega_n \gamma_{1n};\ A_{3n} = -\gamma_{1n};$$

$$A_{4n} = \mu_n \delta \left[\frac{\overline{W}_1 L_1}{2H_w} + \frac{\overline{W}_2 L_2}{2H_w} \right];$$

$$A_{5n} = -2\xi_n \omega_n \gamma_{2n};\ A_{6n} = -\gamma_{2n};$$

$$A_{7n} = \mu_n \delta \left[\frac{\overline{W}_2 L_2}{2H_w} + \frac{\overline{W}_3 L_3}{2H_w} \right];$$

$$A_{8n} = -2\xi_n \omega_n \gamma_{3n};\ A_{9n} = -\gamma_{3n};$$

$$A_{10n} = \mu_n \delta \left(\frac{\overline{W}_3 L_3}{2H_w} \right);$$

$$A_{11n} = -2\xi_n \omega_n \gamma_{4n};\ A_{12n} = -\gamma_{4n};$$

$$A_{13n} = \mu_n \delta;\ A_{14n} = -\mu_n \delta$$

$$A_{13n} = \mu_n \delta;\ A_{14n} = -\mu_n \delta$$

where

$$\delta = \frac{E_c A_c g}{L_e H_w};\gamma_{jn} = \frac{\sum_{i=1}^{3} \overline{W}_i \int_0^{L_i} g_{ji}(x_i)\psi_n(x_i)dx}{\sum_{i=1}^{3} \overline{W}_i \int_0^{L_i} \psi_n^2(x_i)dx} \quad j = 1,2,3,4$$

and

$$\mu_n = \frac{\sum_{i=1}^{3} \overline{W}_i \int_0^{L_i} \psi_n(x_i)dx}{\sum_{i=1}^{3} \overline{W}_i \int_0^{L_i} \psi_n^2(x_i)dx}$$

$\overline{W}_i$ is the dead load per unit length of the i^{th} span of the suspension bridge.

Appendix II

The matrix [A] is

$$[A] = \begin{bmatrix} A_{f1} & & & & & \\ & A_{fj} & & & & \\ & & A_{f12} & & & \\ A_1^1 & & A_{12}^1 & A_{f1} & & \\ & A_j^n & & & A_{nn} & \\ A_1^M & & A_{12}^M & & & A_{MM} \end{bmatrix}$$

where

$$A_{fj} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_{Sj}^2 & -2\xi_{Sj}\omega_{Sj} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\omega_{Sj}^2 & -2\xi_{Sj}\omega_{Sj} & -\omega_{Sj}^2 & -2\xi_{fj}\omega_{fj} \end{bmatrix}_{(4\times4)}$$

$$A_{nn} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi_n\omega_n \end{bmatrix}_{(2\times2)}$$

$$\left[A_j^n\right] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\alpha_{jn}\omega_{Sj}^2 & -2\alpha_{jn}\xi_{Sj}\omega_{Sj} & \gamma_{jn} - \alpha_{jn}\omega_{fj}^2 & \beta_{jn} - 2\alpha_{jn}\xi_{fj}\omega_{fj} \end{bmatrix}$$

and the Matrix [B] can be written as

$$[B] = \begin{bmatrix} B_1 \\ \hdashline B_2 \\ \hdashline B_3 \\ \hdashline Zero \end{bmatrix}_{(3(16)+2\times M)\times(12)}$$

where the submatrices B_1, B_2 and B_3 can be written as

[B1]=

0											
-1											
0											
0											
	0										
	-1										
	0										
	0										
		0									
		-1									
		0									
		0									
			0								
			-1								
			0								
			0								

[B2]=

				0				0			
				-c				s			
				0				0			
				0				0			
					0				0		
					-c				s		
					0				0		
					0				0		
						0				0	
						-c				s	
						0				0	
						0				0	
							0				0
							-c				s
							0				0
							0				0

[B3]=

				0				0			
				-s				-c			
				0				0			
				0				0			
					0				0		
					-s				-c		
					0				0		
					0				0		
						0				0	
						-s				-c	
						0				0	
						0				0	
							0				0
							-s				-c
							0				0
							0				0

The matrices $[A]^j$ and $[B]^j$

$$[A]^j = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_{Sj}^2 & -2\xi_{Sj}\omega_{Sj} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_{Sj}^2 & -2\xi_{Sj}\omega_{Sj} & -\omega_{fj}^2 & -2\xi_{fj}\omega_{fj} \end{bmatrix}$$

$$[B]^j = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, j = 1, 2, 3, 4$$

$$[B]^j = \begin{bmatrix} 0 & 0 & 0 \\ -c & s & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, j = 5, 6, 7, 8$$

$$[B]^j = \begin{bmatrix} 0 & 0 & 0 \\ -s & -c & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, j = 9, 10, 11, 12$$