Reliability Analysis of Fan Type Cable Stayed Bridges Against First Passage Failure under Earthquake Forces

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ABSTRACT: *A reliability analysis of fan type cable stayed bridges against first passage failure under earthquake forces is performed using the method of crossing analysis and the basic theory of reliability. Failure of the bridge deck being the point of interest, the bridge is modeled as a beam supported on springs at different points. The stiffnesses of the springs are determined by a separate 2D static analysis of cable-tower-deck system. The analysis provides a coupled stiffness matrix for the spring system. Using a frequency domain spectral analysis, the power spectral density functions of bending moments at different points of the deck are obtained. Using the first few moments of the power spectral density function, the crossing analysis is carried out to obtain the conditional probability of first passage failure of the bridge deck for a given earthquake ground motion intensity (expressed as r.m.s. ground motion and related to the magnitude of earthquake by an empirical relationship). Probability of occurrence of different magnitudes of earthquake is then combined with the conditional probability of failure to obtain the reliability of the bridge deck against first passage failure. A three span double plane symmetrical fan type cable stayed bridge of total span 689.0 m, is used as an illustrative example. The reliability against first passage failure of the bridge deck is obtained under a number of parametric variations.*

Keywords: First passage failure; Reliability analysis; Crossing analysis, Fan type cable stayed bridges

1. Introduction

The first passage failure means determining the probability that a prescribed threshold level (displacement, stress or other response level) will be exceeded, for the first time, during a fixed period of time. The first passage failure does not lead to the catastrophic failure of a structure, but in view of serviceability consideration it is important. Therefore, the purpose of designing structures against first passage failure is to reduce the probability of such failure, over the expected lifetime of structures, to an acceptable level. In most of the random vibration problems, there is a probability close to unity that any given high response threshold level will be exceeded if the structure is excited for a long enough period of time. Vanmarcke [24] dealt with the problem of the probability of first passage beyond a threshold value by a time dependent random process. Chern [27] dealt with the reliability of a bilinear hysteretic system, subjected to a random earthquake motion, considering first excursion beyond a specified barrier. Solomon and Spanos [22] studied the structural reliability against first passage failure due to seismic excitation. Engelund et al [12] discussed the methods for calculating the approximations of the first passage probability of differentiable non-narrow band processes based on higher order threshold crossings.

Yimin et al [25] studied first passage of uncertain single degree of freedom non-linear oscillators subjected to random excitation. Bayer and Bucher [5] dealt with the reliability assessment of mechanical structures subjected to random excitation using first passage failure analysis. Gan and Zhu [13] studied the first passage failure of multi-degree of quasi-non-integrable-Hamiltonian systems with Gaussian white noise excitations. Au and Beck [4] studied the failure region of the first excursion reliability problem for linear dynamical systems subjected to Gaussian white noise excitation. Not many studies are reported on the reliability analysis of specialty structures like dams, long span bridges, towers, important buildings, etc. for earthquake forces. Such studies are important for two reasons. Firstly, future earthquakes are probabilistic in nature and therefore, a preliminary estimate of the reliability of such structures for future earthquakes must be known using some simplified analysis. Secondly, the different important parameters associated with the earthquake like effect of local soil condition, spatial correlation of ground motion etc. on the seismic reliability estimate of such structures must be evaluated. There have been some studies on the seismic risk analysis of dams [10, 26], pipelines [19], and nuclear structures [6, 21] reported in the past. But very few studies have been carried on seismic reliability analysis of cable supported bridges. Konishi [16] studied the safety and reliability of suspension bridges under wind and earthquake actions. Malla [18] presented a reliability of the cable-system of a cable stayed bridge under stochastic earthquake loading. Pourzeynali and Datta [20] studied the reliability analysis of suspension bridges against flutter failure using basic theory of reliability. Imai and Frangopol [15] studied the system reliability of suspension bridges under different loading and damage scenarios.

In the present paper, a simplified reliability analysis of cable stayed bridge against first passage failure due to seismic forces is performed using the method of crossing analysis. Seismic response of cable stayed bridge due to the random ground motion is obtained using frequency domain spectral analysis through a simplified approach that duly consider the quasi-static response of the bridge deck produced due to the support motion. The probability of first passage failure for future earthquakes is presented for different threshold values of stresses. A parametric study is performed to show the effect of some important parameters such as threshold level, soil condition, degree of correlation of ground motion, and ratio of the components of ground motion on the reliability of cable stayed bridge against the first passage failure.

2. Assumptions

The following assumptions are made for the first passage failure analysis:

i The response process is stationary Gaussian with zero mean value;

ii The evaluated response is always positive (consistent with the spectral analysis), therefore, the single barrier level (called type B barrier, according to Crandall et al [11]) is used;

iii The structure is assumed to be linear and lightly damped;

iv The bridge deck (girder) and the towers are assumed to be axially rigid;

v The bridge deck, assumed as continuous beam, does not transmit any moment to the towers through the girder-tower connection;

vi Cables are assumed to be straight under high initial tension due to the dead load and well suited to support negative force increment during vibration without losing its straight configuration;

vii Beam-column effect is included in the stiffness formulation of the beam by considering only the constant part (non fluctuating component) of axial force.

3. Response Analysis of Fan Type Cable Stayed Bridges

3.1. Seismic Input

Seismic input to the bridge is the power spectral density function (*psdf*) of ground motion. The *psdf* of the ground acceleration described by Clough and Penzien [8] is considered in the study:

$$S_{\ddot{f}_g \ddot{f}_g} = |H_1(i\omega)|^2 \, |H_2(i\omega)|^2 \, S_o \tag{1}$$

in which S_o is the spectrum of the white noise bedrock acceleration; $|H_1(i\omega)|^2$ and $|H_2(i\omega)|^2$ are the transfer functions of the first and the second filters representing the dynamic characteristic of the soil layers above the bedrock, where

$$|H_1(i\omega)|^2 = \frac{1+\left(2\xi_g\,\omega/\omega_g\right)^2}{\left[1-\left(\omega/\omega_g\right)^2\right]^2+\left(2\xi_g\,\omega/\omega_g\right)^2} \tag{2}$$

$$|H_2(i\omega)|^2 = \frac{(\omega/\omega_f)^4}{\left[1-(\omega/\omega_g)^2\right]^2 + (2\xi_g\,\omega/\omega_g)^2} \tag{3}$$

in which ω is the frequency; $\omega_g, \xi_g, \omega_f, \xi_f$ are filtered characteristics; ω_g, ξ_g are the frequency and damping ratio of the first filter representing the bottom layer of the soil, and ω_f, ξ_f are those of the second filter representing the top layer of the soil. The cross spectrum between the random ground motions $\ddot{f}_{g_i}$ and $\ddot{f}_{g_j}$ at two stations *i* and *j* is described given by Hindy and Novak [14] as

$$S_{\ddot{f}_{g_i}\ddot{f}_{g_h}}(r_{ij},\omega) = S_{\ddot{f}_g\ddot{f}_g}(\omega)\rho_{ij}(\omega) \tag{4}$$

in which $S_{\ddot{f}_g\ddot{f}_g}(\omega)$ local spectrum of ground acceleration is given in Eq.(1) which is assumed to be the same for all supports and $\rho_{ij}(\omega)$ is the cross correlation function (coherence function) of the ground motion between two excitation points *i*, *j* and is represented by

$$\rho_{ij}(\omega) = exp\left[-c\left(\frac{r_{ij}\,\omega}{2\pi V_s}\right)\right] \tag{5}$$

in which r_{ij} is the separation distance between stations *i* and *j* measured in the direction of wave propagation; *c* is a constant depending upon the distance from the epicenter and the inhomogeneity of the medium; V_s is the shear wave velocity of the soil; and ω is the frequency (*rad/sec*) of the ground motion. For one sided spectrum it is well known that

$$\sigma_{\ddot{f}_g}{}^2 = S_o\left[\int_0^\alpha |H_1(i\omega)|^2 * |H_2(i\omega)|^2 d\omega\right] \tag{6}$$

$\sigma_{\ddot{f}_g}{}^2$ is the variance of ground acceleration. The empirical relation between the standard deviation of peak ground acceleration and earthquake intensity I_s is taken as Clough and Penzien [8]

$$\sigma_{\ddot{f}_g}{}^2 = 10^{(I_s/3-0.5)}/K^* \tag{7}$$

Also, an empirical relationship between intensity and magnitude of earthquake is taken as Clough and Penzien [8]

$$I_s = (M - 1.3)/0.6 \tag{8}$$

where *M* is the magnitude of earthquake and *K** is a peak factor given by

$$K^* = K' + \frac{0.5772}{K'}; \quad K' = \sqrt{2\,ln(N_0 T)} \tag{9}$$

in which N_0 is the mean rate of zero crossing and is given by

$$N_0 = \frac{1}{2\pi}\sqrt{\int_0^\alpha \omega^2 S_{\ddot{f}_g}(\omega)d\omega \Big/ \int_0^\alpha S_{\ddot{f}_g}(\omega)d\omega} \tag{10}$$

By defining the filter characteristics $\omega_g, \xi_g, \omega_f, \xi_f$ and specifying a standard deviation of the ground acceleration $\sigma_{\ddot{f}_g}$ which can be related to the magnitude of earthquake, the *psdf* of the ground acceleration can be completely defined. The *psdfs* $S_{f_g f_g}(\omega)$ and $S_{\dot{f}_g\dot{f}_g}(\omega)$ of the ground displacement and velocity respectively are related to $S_{\ddot{f}_g\ddot{f}_g}(\omega)$ by

$$S_{f_g f_g}(\omega) = S_{\ddot{f}_g\ddot{f}_g}(\omega)/\omega^4 \tag{11}$$

$$S_{\dot{f}_g\dot{f}_g}(\omega) = S_{\ddot{f}_g\ddot{f}_g}(\omega)/\omega^2 \tag{12}$$

The ground motion is represented along the three principal directions (*u*, *v*, *w*) by defining ratio R_u, R_v and R_w along them such that

$$\ddot{u}_g(t) = R_u\,\ddot{f}_g(t); \quad \ddot{v}_g(t) = R_v\,\ddot{f}_g(t); \quad \ddot{w}_g(t) = R_w\,\ddot{f}_g(t) \tag{13}$$

and the *psdfs* of the ground acceleration in the principal directions of the ground motion (*u*, *v* ,*w*) can be defined as

$$S_{\ddot{u}_g\ddot{u}_g}(\omega) = R_u{}^2 S_{\ddot{f}_g\ddot{f}_g}(\omega); \quad S_{\ddot{v}_g\ddot{v}_g}(\omega) = R_v{}^2 S_{\ddot{f}_g\ddot{f}_g}(\omega); \quad S_{\ddot{w}_g\ddot{w}_g}(\omega) = R_w{}^2 S_{\ddot{f}_g\ddot{f}_g}(\omega) \tag{14}$$

$$\sigma_{\ddot{u}_g}{}^2 = R_u{}^2\sigma_{\ddot{f}_g}{}^2; \quad \sigma_{\ddot{v}_g}{}^2 = R_v{}^2\sigma_{\ddot{f}_g}{}^2; \quad \sigma_{\ddot{w}_g}{}^2 = R_w{}^2\sigma_{\ddot{f}_g}{}^2 \tag{15}$$

The ground motion (α) is defined with respect to the principal direction of the bridge as the angle of inclination between the direction of major component of ground motion with the major direction of the bridge (*x*), see Figure (3). The ground motions along the principal directions of the bridge (*x*, *y*, *z*) are defined as

$$\ddot{x}_g(t) = \ddot{u}_g(t)cos\alpha - \ddot{w}_g(t)sin\alpha \tag{16}$$

$$\ddot{z}_g(t) = \ddot{u}_g(t)\sin\alpha - \ddot{w}_g(t)\cos\alpha \tag{17}$$

$$\ddot{y}_g(t) = \ddot{v}_g(t) \tag{18}$$

The *psdfs* of the ground accelerations along *x*, *y*, *z* can be written as

$$S_{\ddot{x}_g\ddot{x}_g} = \cos^2\alpha S_{\ddot{u}_g\ddot{u}_g} + \sin^2\alpha S_{\ddot{w}_g\ddot{w}_g} = R^2{}_x S_{\ddot{f}_g\ddot{f}_g} \tag{19}$$

$$S_{\ddot{z}_g\ddot{z}_g} = \sin^2\alpha S_{\ddot{u}_g\ddot{u}_g} + \cos^2\alpha S_{\ddot{w}_g\ddot{w}_g} = R^2{}_z S_{\ddot{f}_g\ddot{f}_g} \tag{20}$$

$$S_{\ddot{y}_g\ddot{y}_g} = S_{\ddot{v}_g\ddot{v}_g} = R_y{}^2 S_{\ddot{f}_g\ddot{f}_g} \tag{21}$$

where R_x, R_y ,and R_z are the ratios of the ground motion along the principal axes of the bridge as

$$R_x{}^2 = R_u{}^2\cos^2\alpha + R_w{}^2\sin^2\alpha \tag{22}$$

$$R_z{}^2 = R_u{}^2\sin^2\alpha + R_w{}^2\cos^2\alpha \tag{23}$$

$$R_y{}^2 = R_v{}^2 \tag{25}$$

where R_u, R_v, and R_w are ratios of the ground motion along the principal directions of the ground motion (*u*, *v*, *w*).

3.2. Distribution Function of the Magnitude of Earthquake

Two types of distribution functions of the magnitude of earthquake are considered in the study.

3.2.1. Exponential Distribution

Exponential distribution function of the magnitude of earthquake is based on the Gutenberg-Ritcher Recurrence law

$$\log \lambda_m = a - b\,m \tag{25a}$$

$$\lambda_m = 10^{a - bm} = \exp(\alpha - \beta m) \tag{25b}$$

where λ_m is the mean annual rate of exceedance of magnitude *m*; *a* is the mean yearly number of earthquakes greater than or equal to zero, and *b* describes the relative likelihood of large or small earthquakes. Eq. (25b) implies that the magnitudes are exponentially distributed. Based on Eq. (25b), the probability density function (*PDF*) is given by

$$P_M(m) = \beta e^{-\beta(m - m_0)} \tag{26}$$

where, β = 2.303*b*, and m_0 is the lower threshold magnitude of earthquake, earthquakes smaller than which are eliminated, and *m* is the magnitude of earthquake.

The cumulative distribution function (*CDF*) of the magnitude of earthquake for exponential distribution is given by the following expression

$$F_M(m) = \{1\text{-}\exp[\text{-}\beta(m\text{-}m_0)]\} \tag{27}$$

3.2.2. Gumbel Type-I Distribution

The cumulative distribution function of the magnitude of earthquake for Gumbel type-I distribution is given by the following expression

$$F(m) = Exp[\text{-}\exp\text{-}\alpha(m\text{-}u)] \tag{28}$$

where α and *u* are the parameters for Gumbel Type-1 distribution given by

$$\overline{M} = u + 0.5772/\alpha \tag{29a}$$

$$\sigma_m^2 = \pi^2/6\alpha^2 \tag{29b}$$

in which $\overline{M}$ and σ_m are the mean and standard deviation of the magnitudes of earthquake respectively.

3.3. Response Analysis

With the *psdfs* of the ground acceleration in the principal direction of the bridge as defined by Eqs. (19) to (21) and the correlation function Eq. (5), the *r.m.s* stresses at the critical section (i.e. pt. 3 as shown in Figure (1)) of the bridge are calculated using a modal spectral analysis. The response analysis requires the determination of quasi-static function for defining the displacement of the bridge deck due to the movement of the supports produced by ground motion. This is obtained by performing a static analysis of the bridge cable system. The equation of motion of the bridge deck in terms of relative displacement is written by considering the portion of the deck between two cable supports as a continuum and by assuming the deck to be supported on springs, which replace the restoring action provided by the cables, see Figure (2). The stiffness of the spring system is determined by a separate analysis. A dynamic stiffness approach is used to obtain the mode shapes and frequencies of the bridge [7].

3.4. Modal Spectral Analysis

The modal analysis for the relative vertical displacement $y(x_r, t)$ for any point in the r^{th} deck segment is given as

$$y(x_r, t) = \sum_{n=1}^{\alpha} \phi_n(x_r)\, q_n(t) \qquad r = 1, 2, ..., N_b \tag{30}$$

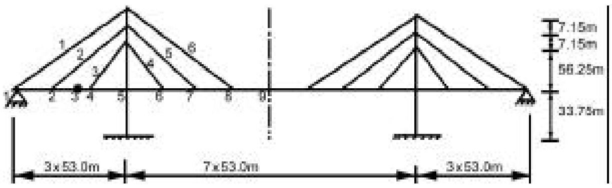

Figure 1. Fan type cable stayed bridge considered for parametric study.

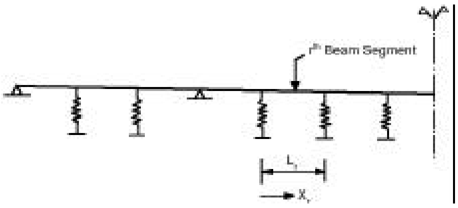

Figure 2. Idealization of the bridge deck.

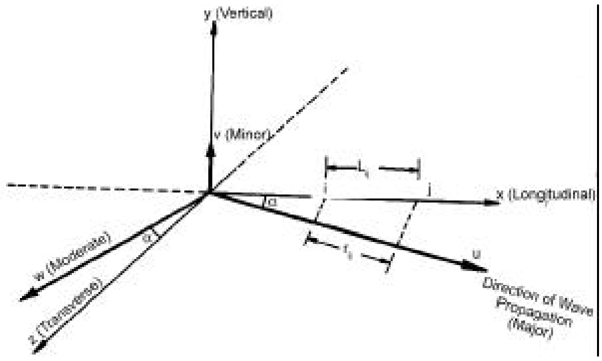

Figure 3. Principal directions of the bridge (x, y, z) and ground motion (u, v, w).

in which $\phi_n(x_r)$ is the n^{th} mode shape of the r^{th} beam segment of the bridge deck and $q_n(t)$ is the n^{th} generalized coordinate. Substituting Eq. (31) into Eq. (30), multiplying by $\phi_m(x_r)$, integrating *w.r.t.* L_r and using the orthogonality of the mode shapes leads to

$$\ddot{q}_n(t) + 2\xi_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = \overline{P}_n(t)$$
$$n = 1, ..., M \tag{31}$$

in which ζ_n and ω_n are the damping ratio and the natural frequency of the n^{th} vertical mode; M is the number of modes considered and $\overline{P}_n(t)$ is the generalized force given as

$$\overline{P}_n(t) = \sum_{j=1}^{8} R_{jn}(x_r)\ddot{f}_j(t) \tag{32}$$

where R_{jn} is the modal participation factor given by

$$R_{jn} = -\frac{\sum_{r=1}^{N_b}\frac{\overline{W}_r}{g}\int_0^{L_r} g_{jr}(x_r)\phi_n(x_r)dx_r}{\sum_{r=1}^{N_b}\frac{\overline{W}_r}{g}\int_0^{L_r}\phi_n^2(x_r)dx_r} \tag{33}$$

in which $g_{jr}(x_r)$, the quasi static function, is the vertical displacement of the r^{th} beam segment of the bridge deck due to unit displacement given in the j^{th} direction of support movement. Following the principles of modal spectral analysis, the expressions for the *psdf* of responses are obtained [1].

$$S_{yy}(x_r,\omega) = \sum_{j=1}^{8}\sum_{k=1}^{8}\sum_{n=1}^{M}\sum_{m=1}^{M}\phi_n(x_r)\phi_m(x_r)\times R_{jn}R_{km}H_n^*(\omega)H_m(\omega)S_{\ddot{f}_j\ddot{f}_k}(\omega) \tag{34}$$

$$S_{gg}(x_r,\omega) = \sum_{j=1}^{8}\sum_{k=1}^{8} g_j(x_r)g_k(x_r)S_{f_jf_k}(\omega) \tag{35}$$

$$S_{yg}(x_r,\omega) = \sum_{j=1}^{8}\sum_{k=1}^{8}\sum_{n=1}^{M}\phi_n(x_r)\times R_{jn}g_k(x_r)H_n^*(\omega)S_{\ddot{f}_j\ddot{f}_k}(\omega) \tag{36}$$

in which $S_{yy}(x_r,\omega)$, $S_{gg}(x_r,\omega)$ and $S_{yg}(x_r,\omega)$ are the *PSDFs* of the relative displacement, the quasi-static displacement and the cross *psdf* between them, respectively.

The expression for the *psdf* of absolute (total) displacement is given by using Eqs. (34), (35) and (36) as

$$S_{yy}(x_r,\omega) = \sum_{j=1}^{8}\sum_{k=1}^{8}\sum_{n=1}^{M}\sum_{m=1}^{M}\phi_n(x_r)\phi_m(x_r) \times R_{jn}R_{km}H_n^*(\omega)H_m(\omega)S_{\ddot{f}_j\ddot{f}_k}(\omega) + 2\sum_{j=1}^{8}\sum_{k=1}^{8}\sum_{n=1}^{M}\phi_n(x_r)R_{jn}g_k(x_r)H_n^*(\omega)S_{\ddot{f}_j\ddot{f}_k}(\omega) + \sum_{j=1}^{8}\sum_{k=1}^{8}g_j(x_r)g_k(x_r)S_{f_jf_k}(\omega) \tag{37}$$

In Eq. (37) $S_{\ddot{f}_j\ddot{f}_k}(\omega)$, $S_{\ddot{f}_j\ddot{f}_k}(\omega)$ and $S_{f_jf_k}(\omega)$ are expressed in terms of the *psdf* of ground acceleration $S_{\ddot{f}_g\ddot{f}_g}(\omega)$ using the coherence function given by Eq. (5) and the ratio between the three components of the ground motion along the global axes of the bridge (R_x, R_y, R_z) as given by Eqs. (22), (23), and (24).

Similar expressions can be obtained for the *psdf* of the bending moment at any point in the r^{th} beam segment of the bridge deck as those derived for the total displacement by replacing $\phi(x_r)$ and $g_{jr}(x_r)$ by and $E_d I_r d^2 \phi(x_r)/dx^2$ respectively. $E_d I_r d^2 g_{jr}(x_r)/dx^2$ is obtained from the quasi-static analysis of the entire bridge using the stiffness approach as mentioned before.

3.5. Statistical Parameters of Response

For obtaining the statistical properties of the response process, the first few moments of the response power spectral density function are needed. The j^{th} moment of the *psdf* of any response may be defined as

$$\lambda_j = \int_0^\alpha \omega^j S_{YY}(\omega)\, d\omega \quad \text{where } j = 0, 1, 2, ... \tag{38}$$

The zeroth and second moments may be recognized as the variances of the response and the first time derivative of the response respectively.

$$\lambda_0 = \int_0^\infty S_{YY}(\omega)\, d\omega = \sigma^2_{YY} \tag{39}$$

$$\lambda_2 = \int_0^\infty \omega^2 S_{YY}(\omega)\, d\omega = \sigma^2_{\dot{Y}\dot{Y}} \tag{40}$$

The mean rate of zero crossing at positive slopes is given by Trifunac and Brady [23]

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0}} \tag{41}$$

Another quantity of interest is the dispersion parameter q given by

$$q = \sqrt{1 - \frac{\lambda^2_1}{\lambda_0 \lambda_2}} \tag{42}$$

The value of q lies between 0 and 1. It can be shown that q is small for a narrow band process and relatively large for a wide band process. The mean rate of crossing a specified level A at a positive slope by a stationary zero mean Gaussian random process $z(t)$ can be expressed by Lin [17]

$$\nu_a = \nu_0 e^{\frac{-\psi^2}{2}} \tag{43}$$

$$\psi = \frac{A}{\sigma_{YY}} \tag{44}$$

It has been confirmed by theoretical as well as simulation studies that the probability of a stationary response process remaining below a specified barrier level decays approximately exponentially with time as given by the relationship [9, 11].

$$L(t) = L_o e^{-aT} \tag{45}$$

where, L_o is the probability of starting below the threshold, α is the decay rate, and T is the duration of the response process.

At high barrier levels, L_o is practically equal to one, and the decay rate is given by the following expressions for processes with double barrier and one sided barrier respectively [17, 24] as

$$a_D = 2\nu_a \tag{46}$$

$$a_S = \nu_a \tag{47}$$

In case of relatively low threshold levels, an improved value can be obtained by using the expressions for the probability of starting below the threshold and the decay rate [24].

$$L_o = 1 - e^{\frac{-\psi^2}{2}} \tag{48}$$

$$a_D{}^* = a_D \frac{1 - e^{-\sqrt{\frac{\pi}{2}}(q\psi)}}{1 - e^{-\psi^2/2}} \tag{49}$$

$$a_S{}^* = a_S \frac{1 - e^{-\sqrt{2\pi}(q\psi)}}{1 - e^{-\psi^2/2}} \tag{50}$$

In Eq. (45), α is replaced by $a_D{}^*$ or $a_S{}^*$ depending upon the problem.

4. Reliability Estimation Against First Passage Failure

For an earthquake with given magnitude M, the probability of first passage failure, i.e., the probability that the response z is larger than a threshold level A, can be determined from the following relationship

$$p\left[z > A \middle| M\right] = 1 - L(t) \tag{51}$$

where T is the duration of the response.

If $f_M(M)$ is the probability density function of earthquake magnitude, the probability of first passage failure, provided that an earthquake occurs, can be calculated from [2].

$$p_E = p[z > A] = \int_{M=M_l}^{M_u} p\left[z > A \middle| M\right] f_M(t)\, dM \tag{52}$$

in which M_1 and M_u are the upper and lower limits of the magnitude of earthquake.

If the rate of earthquake occurrence for the seismotectonics region considered in the study is a constant and n is the average number of earthquakes per year in the magnitude range of interest for the source region (assuming the events to be independent), the probability of at least one failure due to earthquake in "m" years can be expressed as

$$P_F = 1 - (1 - p_E)^{mn} \qquad (53)$$

5. Numerical Study

A double plane symmetrical cable stayed bridge, used as an illustrative example by Au et al [3] and shown in Figure (1) is considered for the parametric study. The connection between the cables and the towers are assumed to be of hinged type. In addition, the following data are assumed for the analysis of the problem, $E_c = E_d$ where E_c and E_d are the modulus of elasticity of the cable and the deck respectively; $\xi = 0.02$ for all modes; and the tower-deck inertia ratio, the ratio between three components of the ground motion (R_u: R_v: R_w) and are taken to be 4, (1.0:1.0:1.0) and 0.0 respectively and duration of earthquake as 15*sec*., unless mentioned otherwise. The random ground motion is assumed to be homogeneous stochastic process which is represented by Clough and Penzien [8] double filter *psdf* given by two sets of filter coefficients representing the soft and firm soils respectively. For the soft soil, the coefficients are taken as $\omega_g = 6.2832 rad/sec$; $\omega_f = 0.62832 rad/sec$; $\xi_g = \xi_f = 0.4$, while those for the firm soils are $\omega_g = 15.708 rad/sec$; $\omega_f = 1.5708 rad/sec$; $\xi_g = \xi_f = 0.6$. The spatial correlation function used in the parametric study is given by Eq. (5) in which the value of c is taken as 2; $Vs = 70 m/sec$ and $Vs = 330 m/sec$ for the soft and firm soils respectively. The *r.m.s* ground acceleration is related to intensity of earthquake by the empirical equation given by Eq. (7). Intensity of earthquake I_s in turn is related to magnitude of earthquake given by Eq. (8). The input for excitation is thus the intensity of earthquake for which $\sigma_{\ddot{u}_g}$ value can be calculated using Eq. (15). From $\sigma_{\ddot{u}_g}$, the value of S_0 required to define the ordinates of the double filter *psdf* can be obtained using Eq. (6). The failure mechanism assumed for determining the probability of failure is considered as the failure of bridge deck section 3-3, shown in Figure (1), where the maximum bending moment occurs.

5.1. Effect of Barrier Level on the Reliability

Effect of the barrier level on the reliability of the bridge is shown in Figure (4). The barrier level is taken as 15%, 20%, 25%, 30%, 33%, 40%, 50% and 70% of the yield stress assuming that the barrier level is the difference between yield stress and the pre-stress in the girder (deck). It is seen from the figures that the reliability increases as the barrier level increases as it would be expected. However, the variation is not linear; it tends to follow an *S* shaped curve. For soft soil condition, the variation of reliability with barrier level may be very steep in the lower range of barrier levels i.e., in the lower range of barrier level, a small decrease in the barrier level may significantly decrease the reliability of soft soil. Further, it is seen from Figure (4) that the reliability for a particular barrier level is higher for firm soil and the difference between the reliabilities obtained for the soft and firm soil conditions considerably increases in the lower range of barrier level. This is the case because the response of the bridge is significantly more for soft soil conditions compared to hard soil condition.

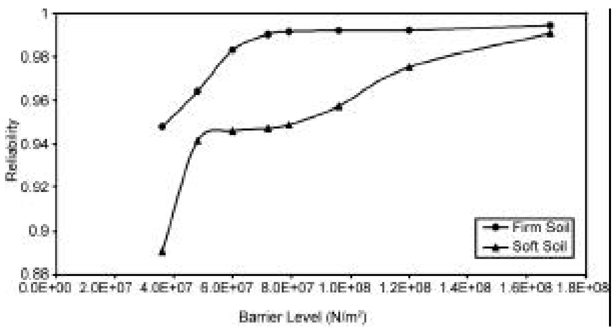

Figure 4. Effect of barrier level on the reliability.

5.2. Effect of Correlation of Ground Motion on the Reliability

Figures (5) and (6) show the variation of reliability with barrier level for three cases of ground motion, i.e. fully correlated, partially correlated and uncorrelated. It is seen from the figures that fully correlated ground motion provides more reliability compared to uncorrelated ground motion. This is the case because uncorrelated / correlated ground motion induces additional bending moment in the deck due to the phase lag of ground motion between different supports. The difference between the reliabilities for the three cases is not very significant for the firm

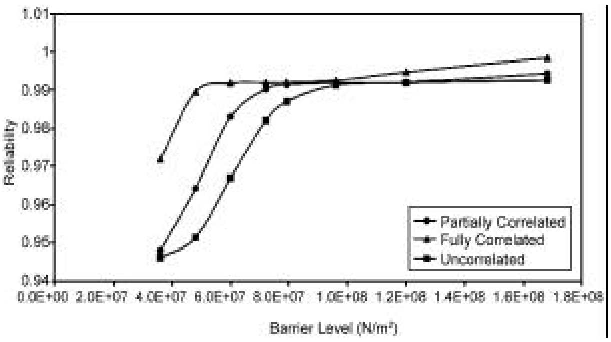

Figure 5. Effect of correlation of ground motion on the reliability (Firm soil).

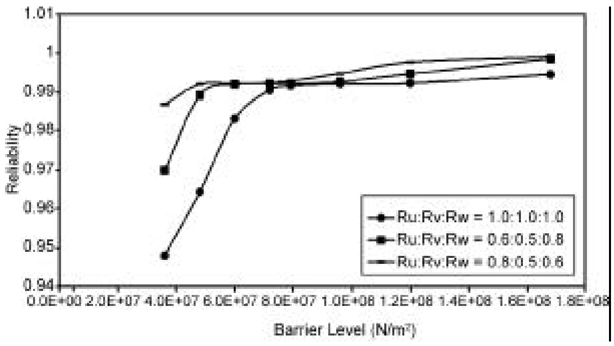

Figure 7. Effect of the ratio of the components of ground motion on the reliability (Firm soil).

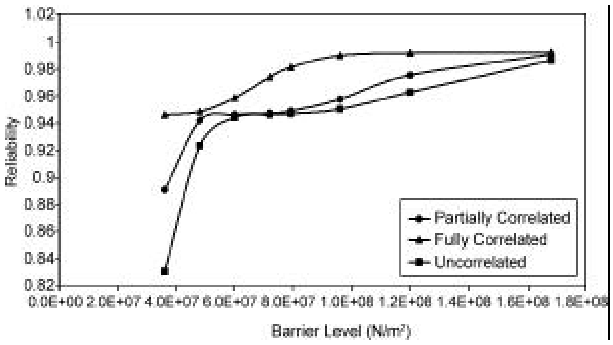

Figure 6. Effect of correlation of ground motion on the reliability (Soft soil).

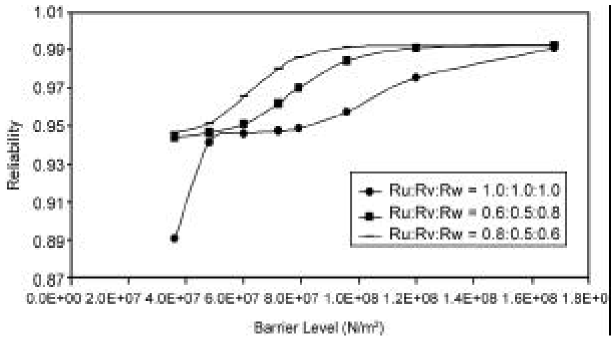

Figure 8. Effect of the ratio of the components of ground motion on the reliability (Soft soil).

soil. However, the difference between them is significant for the soft soil, see Figure (6). Further, the difference between the reliabilities is considerably reduced at the higher end of the barrier level.

5.3. Effect of the Ratio of the Components of Ground Motion on the Reliability

The effect of this ratio on the variation of reliability with the barrier level is shown in Figures (7) and (8). It is seen from the figures that the ratio has an effect on this variation, especially near the lower end of the barrier level. For the soft soil condition, the difference between the reliability estimates for different ratios of components of ground motion is more. It is to be noted that not only the increase in the vertical component of ground motion increases the probability of failure of the bridge deck, but also the increase in horizontal component of ground motion does the same. This is the case because increase in the horizontal component of ground motion increases the deck response due to the presence of quasi-static component of response in the total response. Thus, the reliability estimate is significantly influenced by the ratio between components of ground motion especially for the soft soil condition.

5.4. Effect of Angle of Incidence on the Reliability

Figures (9) and (10) show the effect of angle of incidence on the variation of reliability with barrier level. Three values of angle of incidence are considered namely, 0^0 i.e. major direction of earthquake along the longitudinal axis of the bridge and the other two cases having 30^0 and 70^0 angles of incidence with the longitudinal axis of the bridge respectively. It is seen from the figures that 0 degree angle of incidence provides less reliability estimate in the lower range of barrier level compared to other two angles of incidences. However, the difference between the reliabilities is not very significant.

5.5. Effect of Magnitude of Earthquake on the Reliability

The effect of the distribution of magnitude of earthquake on the reliability is shown in Figure (11). The figure shows the variation of reliability with barrier level for the two distributions of the magnitude of earthquake (i.e. exponential and gumbel distributions). It is seen from the figures that the reliability increases with the increase in beta values for the exponential distribution. The difference between

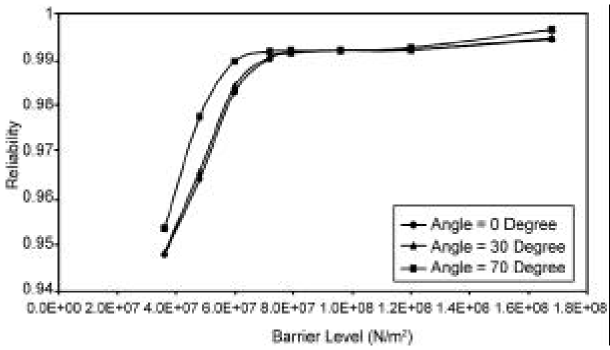

Figure 9. Effect of angle of incidence on the reliability (Firm soil).

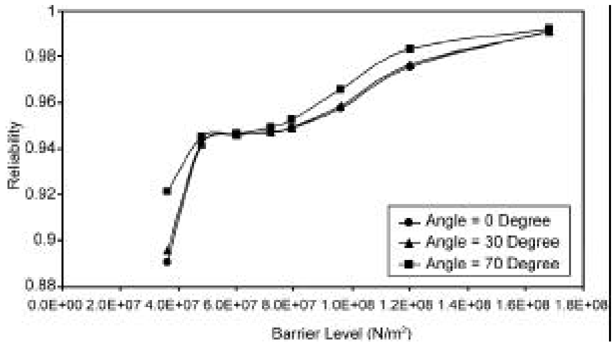

Figure 10. Effect of angle of incidence on the reliability (Soft soil).

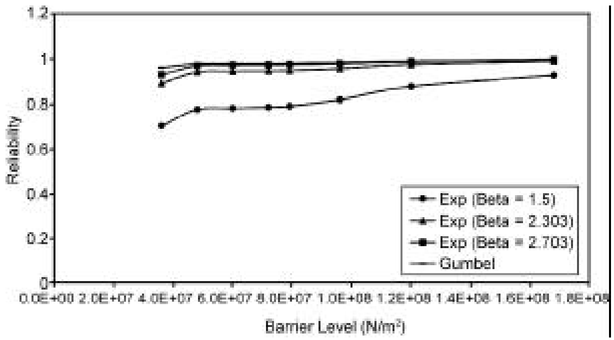

Figure 11. Effect of distribution of the magnitude of earthquake on the reliability (Firm soil).

reliabilities obtained for two beta values is considerably more for lower values of barrier level. Above a certain value of beta, the reliability nearly approaches unity for all barrier levels. Further, Gumbel distribution provides much higher value of reliability as compared to the exponential distribution (for beta = 1.5). For the soft soil condition, see Figure (12), the effect of the distribution of the magnitude of earthquake is more pronounced.

5.6. Effect of the Average Number of Earthquake on the Reliability

Figure (13) shows the variation of reliability with the

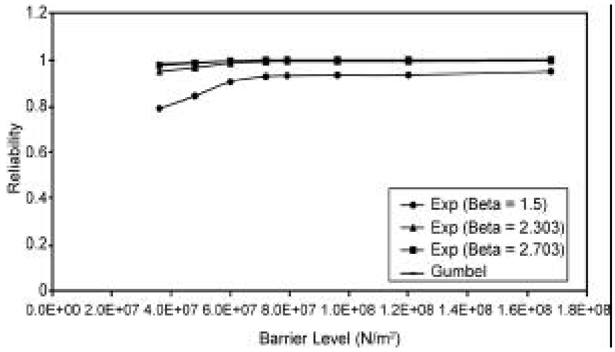

Figure 12. Effect of distribution of the magnitude of earthquake on the reliability (Soft soil).

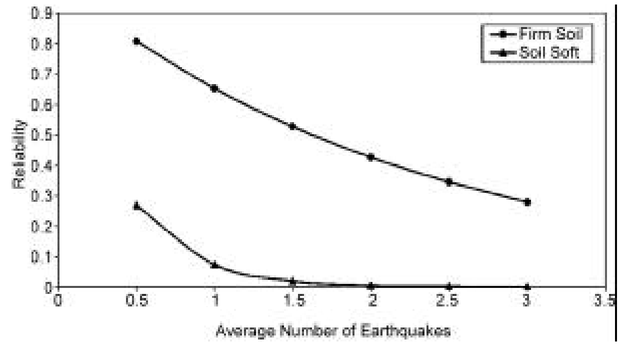

Figure 13. Variation of reliability with average number of earthquake per year (for a barrier level of 33%).

average number of earthquakes per year for a barrier level of 33% of the yield stress. It is seen that the reliability decreases with the increase of average number of earthquake per year, as it would be expected. The variation is nonlinear and more steep for the soft soil condition. For an average number of earthquake of 0.5 per year, the reliability could be as low as 0.27 for soft soil condition.

5.7. Effect of the Duration of Earthquake on the Reliability

Figure (14) shows the variation of reliability with the duration of earthquake for a barrier level of 33%. It

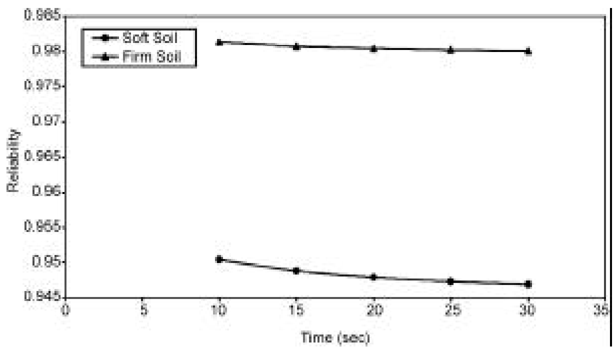

Figure 14. Variation of reliability with duration of earthquake (for a barrier level of 33%).

is seen from the figures that reliability decreases mildly with the increase of the duration of earthquake for both soft and firm soil conditions. Thus, the duration of earthquake does not have very significant influence on the reliability.

6. Conclusions

Reliability against first passage failure of cable stayed bridges under earthquake excitation is presented. The responses of cable stayed bridges are obtained for random ground motion which is modeled as stationary random process represented by double filter power spectral density function and a correlation function. The responses are obtained by frequency domain spectral analysis. Conditional probability of first crossing the threshold level for a given *r.m.s* ground acceleration is obtained by using the moments of the *psdf* of the response. The *r.m.s* value of the ground acceleration is related to the intensity and the magnitude of earthquake by empirical equations, and the probability density function of the earthquake is combined with the conditional probability of failure to find the probability of first passage failure. Using the above method of analysis, a cable stayed bridge is analyzed and the probability of first passage failure is obtained for a number of parametric variations. The results of the numerical study lead to the following conclusions:

- ❖ The reliability against first passage failure increases sharply with the increase in barrier level in the lower range of its values.
- ❖ For the soft soil condition, the reliability is considerably less as compared to the firm soil condition; the difference between the two increases in the lower range of barrier level.
- ❖ Uncorrelated ground motion provides lower estimates of the reliability as compared to the fully correlated ground motion. The difference is significantly more for the soft soil conditions.
- ❖ The increase in both vertical and horizontal components of ground motions decreases the reliability estimates. For soft soil condition, this effect is more pronounced.
- ❖ Angle of incidence of earthquake with respect to the longitudinal axis of the bridge does not have considerable effect on the reliability estimates in the lower end of the barrier level.
- ❖ Gumbel distribution of the magnitude of the earthquake provides a higher estimate of reliability and gives values close to those obtained by the exponential distribution for high values of the parameter β.
- ❖ The duration of ground motion does not have significant influence on the reliability estimates.

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