Seismic Wave Propagation in a Multi-Layered Geological Region, Part II: Transient Case

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ABSTRACT: *The Two-dimensional problem of the transient wave propagation in elastic multi-layered half-space is studied by the Direct Boundary Integral Equation Method (DBIEM) combined with the finite difference procedure applied to the time variable. By means of the Wilson-θ method the equations of motion are transformed into a set of elliptic partial differential equations, and then, the DBIE-procedure is applied. The present hybrid formulation employs the fundamental solution depending neither on the frequency nor on the time variable. This is the main advantage of the proposed method. The theoretical seismograms in the time domain are obtained on the free surfaces of two real geological situations for a multi-layered soil region with existence of salt ore deposits.*

Keywords: Multi-layered geological region; Transient wave problem; DBIEM together with Wilson - θ method

1. Introduction

Transient waves are generated by the body force *f(r, t)* in the Navier-Cauchy equation of motion for elastic solids, or the surface displacement *u(r,t)*, or traction *p(r,t)* given as boundary conditions for the corresponding boundary-value problem. If the time function for these sources is time-harmonic and the motion is observed long after the initiation of the source, the wave motion is also harmonic in time and it is called steady state. Otherwise, the wave motion is transient. In an infinite medium, the transient wave field generated by a concentrated force of arbitrary time function was determined by G. Stokes in 1849. He constructed the general solution of the inhomogeneous Navier-Cauchy equation of motion by what is now called the method of retarded potentials. Next in mathematical complexity is the problem of transient waves in a half-space. H. Lamb in 1904 investigated this problem first. The "half-space" and related problems have since become a focal point for many studies. Many applications of elastodynamics in seismic mechanics, design of earthquake-resistant structures, dynamics of structural foundations as well as basic studies on dynamics of material defects begin with the model of a half-space. The solution is much more complicated when additional plane boundaries are introduced to form a layered half-space. This is a basic model for theoretical seismograms, which has always been a subject of intensive study.

The main aims of this paper are:

- ❖ To find the solution of a two-dimensional plane-strain transient wave propagation problem for a multi - layered geological region with complex geometry on the base of a hybrid usage of both finite differences scheme of Wilson - θ method together with the BIEM.
- ❖ To show that the changes in the soil region during the years of the exploitation process lead to the change in its dynamic response, i.e. to the change in the obtained theoretical seismograms.

The propagation of transient elastic waves through the layered half-space is of considerable interest to engineers, geologists and seismologists. Lacking any analytical method to treat such complex problems, resort has been made to the numerical techniques-finite element method and boundary integral equation method. Most of the earlier works on transient wave propagation by *BEM* involved transform domain formulations in conjunction with numerical inversion scheme [8-12], etc. The direct time-domain formulation of the *BIEM* is used in [13-16], etc. Detail analysis for advantages and disadvantages of *BIEM* in comparison with other numerical methods is given in [16]. In this paper a *BIEM* formulation, different from the above-cited two formulations, is proposed. By means of the finite difference method, applied to the time variable, the equations of motion are transformed into a set of elliptic partial differential equations, and then, the *DBIE*-procedure is applied at every time step.

The most of the works are devoted to the multi-layered regions with simple geometry of the boundary between layers (usually parallel boundaries). To the authors opinion there is a lack of studies involving real multi-layered regions with complex geometry of the boundaries between layers (non-parallel boundaries).

It is given here BIEM formulation, novel numerical scheme and FORTRAN codes created for solution of a 2D transient seismic waves propagation problem in a multi-layered soil region with very complex geometry of the soil layers and this geological column is a real geological region in East Bulgaria.

2. Formulation of the Problem

2.1. Governing Equations

The mathematical theory of elasticity is formulated in terms of body force f_i, surface force (traction) P_i, stress tensor σ_{ij}, strain tensor ε_{ij} and displacement vector u_i of an elastic body. The isotropic materials are characterized by material constants such as the shear module *G*, Lame's constants λ, μ and mass density ρ. The linear theory of elasto-dynamics is embodied in the following set of governing equations for a body of volume *V* enclosed by a surface $S = S_u + S_p$

Equation of motion

$$\sigma_{ij,j} + f_i = \ddot{u}_i \quad x \in V \tag{1}$$

Physical equations in elastic case

$$\sigma_{ij} = 2\mu(v/(1-2v))\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij} \quad x \in V \tag{2}$$

here: $v = \dfrac{\lambda}{2(\lambda+\mu)}$

Equation of geometry

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

The governing equations of the two-dimensional motion of an isotropic and homogeneous elastic medium are obtained by Eqs. (1), (2) and (3) and have the form

$$\left(C_p^2 + C_s^2\right)u_{i,ij} + C_s^2 u_{j,ii} - \ddot{u}_j = -\rho^{-1} f_j(t) \tag{4}$$

where *i*, *j* = *x*, *y*, the summation convention applies to the repeated suffix *i*, the notation $(.)_{,i} = \partial(.)/\partial x_i$ is introduced to denote the partial derivatives with respect to co-ordinates x_i, the dot signifies material time differentiation and $C_p = [(\lambda + 2\mu)/\rho]^{1/2}$, $C_s = (\mu/\rho)^{1/2}$ are speeds of the longitudinal *P*- and shear *S*- wave, respectively. The problem is two-dimensional and of plane strain and $u_x(x,y,t)$, $u_y(x,y,t)$ denote the horizontal and the vertical displacement components respectively.

The initial and boundary conditions are

$$u_i(x,y,t) = \bar{u}_i \ \text{ at } (x,y) \in S_u, \sigma_{ij} n_j = \bar{p}_i \ \text{ at } (x,y) \in S_p$$

$$u_i(x,y,t_0) = u_{i0}, \ \dot{u}_i(x,y,t_0) = \dot{u}_{i0} \quad (x,y) \in (x,y) \in S_u \tag{5}$$

The governing equations (4) together with the boundary and initial conditions (5) present the boundary-value problem of the transient elasto-dynamics.

2.2. BIE Formulation of the Problem

The considered problem can be solved using the following approaches:

❖ Laplace or Fourier transformation leading to the next BIE

$$c_y \tilde{u}_i(r,\omega) = \int_S \left[U_{ij}^*(r,\eta,\omega)\tilde{p}_j(\eta,\omega) - P_{ij}^*(r,\eta,\omega)\tilde{u}_i(\eta,\omega)\right] dS$$
$$+ \int_V U_{ij}^*(r,\eta,\omega) Q_j(\eta,\omega) dV \tag{6}$$

Here *r* and η are the position vectors of the field and running points; the constants c_{ij} depend on the geometry of the boundary at the collocation point *r*; U_{ij}^* and P_{ij}^* are the fundamental solutions of the displacement and the traction given in the Appendix; $\tilde{u}_i$, $\tilde{p}_i$ are the corresponding Laplace or Fourier transformations of the displacement and the traction; Q_j is the sum of the Laplace or Fourier transformations of f_j and the members, containing the initial velocity and displacement; the Laplace transformation variable is $s = -i\omega$. This method is used by the authors in [1, 2, 3].

❖ A hybrid method consists of two stages:

i) Application of the method of finite differences in respect to the time variable-then the set of parabolic partial differential equations is transformed into a set of elliptic partial differential equations;

ii) The *BIEM* is applied at each time step. This hybrid formulation is used by authors in [4] in the simple case of an elastic half-space. This approach will be used in the current paper but in the case of a multi-layered geological region with non-parallel boundaries of the layers. Due to the fact that this method is applied here in a multi-layered region with complex geometry, we will describe it in the next section.

3. A Hybrid Method of Finite Differences Method and BIEM for the Solution of the Transient Elasto-Dynamic Problem

The application of the Wilson-θ method [5] to $\dot{u}_i$ and $\ddot{u}_i$ yields

$$\dot{u}_i(t+\theta\Delta t)=\frac{3}{\theta\Delta t}[u_i(t+\theta\Delta t)-u_i(t)]-2\dot{u}_i(t)-\frac{\theta\Delta t}{2}\ddot{u}(t) \quad (7)$$

$$\ddot{u}_i(t+\theta\Delta t)=\frac{6}{\theta^2\Delta t^2}[u_i(t+\theta\Delta t)-u_i(t)]-2\ddot{u}_i(t)-\frac{6}{\theta\Delta t}\dot{u}(t)$$

where Δt denotes the time step and θ is a coefficient, which secures the stability and the convergence of the finite difference procedure. This procedure becomes unconditionally stable for $\theta > 1.37$. After substitution of Eq. (7) in Eq. (4), the next system of elliptic linear partial differential equations is obtained at time $t+\theta\Delta t$

$$\left(C_P^2-C_S^2\right)u_{j,ji}(t+\theta\Delta t)+C_S^2u_{i,jj}(t+\theta\Delta t)-k^2u_i(t+\theta\Delta t)=-X_i \quad (8)$$

where

$$k^2=\frac{6}{\theta^2\Delta t^2};\ X_i=\rho^{-1}f_i(t+\theta\Delta t)+\frac{6}{\theta^2\Delta t^2}u_i(t)+\frac{6}{\theta\Delta t}\dot{u}_i(t)+2\ddot{u}_i(t)$$

As it can be seen from Eqs. (8) the last are the same as the elliptic partial differential equations, obtained after the application of the Laplace transform with $k^2=6/\theta^2(\Delta t)^2$. That is why the conventional direct *BIEM* can be applied to Eqs. (8) at a given time step Δt. The fundamental solutions, given in the Appendix, can be used, but in this case $s=-i\omega=k$. The discretization used in the analysis involves *N* constant boundary elements and *M* constant triangular elements. The points, where the unknown values are considered, are called "nodes" and they are in the middle of the element for the so-called "constant" element. The values of the displacement and the traction are assumed to be constant over each element and equal to the value at the mid-element node. For this type of elements the boundary is always "smooth" as the node is at the centre of the element. The discretization nodes are in the middle points of the all *N* boundary elements over the free surface and they are in the centres of gravity of the all *M* constant triangular elements used for discretization of the half-space. As a result the next system of discretized boundary integral equations is obtained

$$\begin{bmatrix} c_{xx} & 0 \\ 0 & c_{yy}\end{bmatrix}\begin{bmatrix} u_x\left(x_o^p,y_0^p\right) \\ u_y\left(x_o^p,y_0^p\right)\end{bmatrix}=\begin{bmatrix} A_x\left(x_o^p,y_0^p\right) \\ A_y\left(x_o^p,y_0^p\right)\end{bmatrix}-\begin{bmatrix} B_x\left(x_o^p,y_0^p\right) \\ B_y\left(x_o^p,y_0^p\right)\end{bmatrix}+\rho f(t)\begin{bmatrix} C_x\left(x_o^p,y_0^p\right) \\ C_y\left(x_o^p,y_0^p\right)\end{bmatrix}+\rho\begin{bmatrix} D_x\left(x_o^p,y_0^p\right) \\ D_y\left(x_o^p,y_0^p\right)\end{bmatrix} \quad (9)$$

where $f(t)$ denotes the time signature of the source, $c_{xx}=c_{yy}=1$ for the interior points, $c_{xx}=c_{yy}=0.5$ for the points, located at the boundary

$$\begin{bmatrix} A_x\left(x_0^p,y_0^p\right) \\ A_y\left(x_0^p,y_0^p\right)\end{bmatrix}=\begin{bmatrix} \sum_{k=1}^{N}p_x\left(x_0^k,y_0^k\right)\int_{\Gamma_k}U_{xx}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Gamma_k+\sum_{k=1}^{N}p_y\left(x_0^k,y_0^k\right)\int_{\Gamma_k}U_{yx}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Gamma_k \\ \sum_{k=1}^{N}p_x\left(x_0^k,y_0^k\right)\int_{\Gamma_k}U_{yx}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Gamma_k+\sum_{k=1}^{N}p_y\left(x_0^k,y_0^k\right)\int_{\Gamma_k}U_{yy}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Gamma_k\end{bmatrix} \quad (9a)$$

$$\begin{bmatrix} B_x\left(x_0^p,y_0^p\right) \\ B_y\left(x_0^p,y_0^p\right)\end{bmatrix}=\begin{bmatrix} \sum_{k=1}^{N}u_x\left(x_0^k,y_0^k\right)\int_{\Gamma_k}P_{xx}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Gamma_k+\sum_{k=1}^{N}u_y\left(x_0^k,y_0^k\right)\int_{\Gamma_k}P_{yx}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Gamma_k \\ \sum_{k=1}^{N}u_x\left(x_0^k,y_0^k\right)\int_{\Gamma_k}P_{yx}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Gamma_k+\sum_{k=1}^{N}u_y\left(x_0^k,y_0^k\right)\int_{\Gamma_k}P_{yy}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Gamma_k\end{bmatrix} \quad (9b)$$

$$\begin{bmatrix} C_x\left(x_0^p,y_0^p\right) \\ C_y\left(x_0^p,y_0^p\right)\end{bmatrix}=\begin{bmatrix} -U_{xx,x_S}^*\left(x_0^p-x_S,y_0^p-y_S\right)-U_{xx,y_S}^*\left(x_0^p-x_S,y_0^p-y_S\right) \\ -U_{yx,x_S}^*\left(x_0^p-x_S,y_0^p-y_S\right)-U_{yy,y_S}^*\left(x_0^p-x_S,y_0^p-y_S\right)\end{bmatrix} \quad (9c)$$

$$\begin{bmatrix} D_x\left(x_0^p,y_0^p\right) \\ D_y\left(x_0^p,y_0^p\right)\end{bmatrix}=\begin{bmatrix} \sum_{k=1}^{M}\alpha_x^k\int_{\Omega_k}U_{xx}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Omega_k+\sum_{k=1}^{M}\alpha_y^k\int_{\Omega_k}U_{xy}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Omega_k \\ \sum_{k=1}^{M}\alpha_x^k\int_{\Omega_k}U_{yx}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Omega_k+\sum_{k=1}^{M}\alpha_y^k\int_{\Omega_k}U_{yy}^*\left(x_0^p-x_0^k,y_0^p-y_0^k\right)d\Omega_k\end{bmatrix} \quad (9d)$$

$$\alpha_i^k=ku_i\left(x_0^k,y_0^k,t\right)+\frac{6}{\theta\Delta t}\dot{u}_i\left(x_0^k,y_0^k,t\right)+2\ddot{u}_i\left(x_0^k,y_0^k,t\right)$$

Note, that Eqs. (9a -9c) are evaluated at time $t+\theta\Delta t$, while Eq. (9d) is evaluated at time *t*. In the above matrices $u_x\left(x_0^p,y_0^p\right)$ and $u_y\left(x_0^p,y_0^p\right)$ are components of the unknown displacement, $p_x\left(x_0^p,y_0^p\right)$ and $p_y\left(x_0^p,y_0^p\right)$ are components of the unknown traction, x_S,y_S are the co-ordinates of the seismic source, and the notations $(.)_{,x_S}=\partial(.)/\partial x_S$, $(.)_{,y_S}=\partial(.)/\partial y_S$ are introduced to denote the partial derivatives with respect to x_S and y_S respectively.

4. Numerical Realization Algorithm

The numerical procedure for the solution of the transient elasto-dynamic problem described above is

- Solve the system of Eq. (9) for $t = 0 + \theta\Delta t$ and obtain u_x, u_y, p_x, p_y at all collocation points on the boundary (note that for $t = 0, \alpha_i^k = 0$).
- For $t = 0 + \theta\Delta t$ evaluate u_x, u_y, p_x, p_y and α_x^k, α_y^k at all interior collocation points.
- For $t = 0 + \Delta t$ by the Wilson-θ method calculate $u_x, u_y, \dot{u}_x, \dot{u}_y, \ddot{u}_x, \ddot{u}_y$.
- For $t = \Delta t + \theta\Delta t$ solve the system of Eq. (9) and obtain u_x, u_y, p_x, p_y at all collocation points on the boundary.
- For $t = \Delta t + \theta\Delta t$ evaluate u_x, u_y, p_x, p_y and α_x^k, α_y^k at all interior collocation points.
- For $t = 2\Delta t$ by the Wilson-θ method calculate $u_x, u_y, \dot{u}_x, \dot{u}_y, \ddot{u}_x, \ddot{u}_y$.
- For $t = 2\Delta t + \theta\Delta t$ solve the system of Eq. (9) and obtain u_x, u_y, p_x, p_y at all collocation points on the boundary.
- For $t = 2\Delta t + \theta\Delta t$ evaluate u_x, u_y, p_x, p_y and α_x^k, α_y^k at all interior collocation points.

Based on the above scheme, the evaluation of responses for $t = 3\Delta t + \theta\Delta t$ and subsequent observation times is straightforward.

5. Numerical Example

Two real geological situations for a multi-layered soil media with existence of salt ore deposits, in Figures (1) and (2) are considered. These situations concern one and the same geological region but in different periods of its exploitation -in 1951, see Figure (1) and 1994, see Figure (2). Due to the symmetry the half of the geometry is given

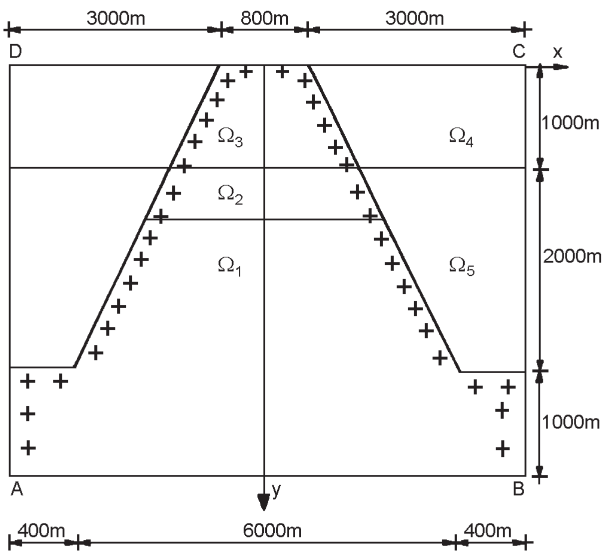

Figure 1. The (2D) geometry of the multi-layered geological region in 1951.

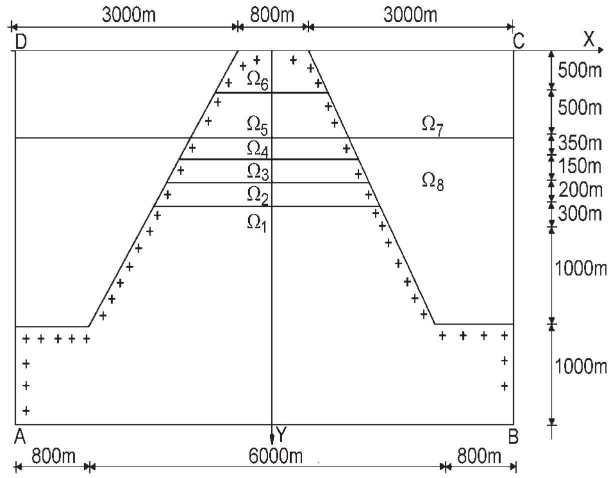

Figure 2. The (2D) geometry of the multi-layered geological region in 1994.

correspondingly in Figures (3) and (4). There is a change of the situation during the years when the exploitation of the salt ore deposits has been done. The main goal is to show that the changes in the soil region during all these years lead to the change in its dynamic response, i.e. to the change in the obtained theoretical seismograms. It is assumed that the region is subjected to the buried explosive seismic load, which time function is the parabolic ramp function given in Figure (5)

$$f(t) = \begin{cases} 0 & for\ t < 0 \\ 0.5t^2 & for\ 0 \le t \le \Delta \\ 0.5t^2 - (t-\Delta)^2 & for\ \Delta \le t \le 2\Delta \\ \Delta^2 & for\ t \ge 2\Delta \end{cases} \qquad \Delta = 0.1s \tag{10}$$

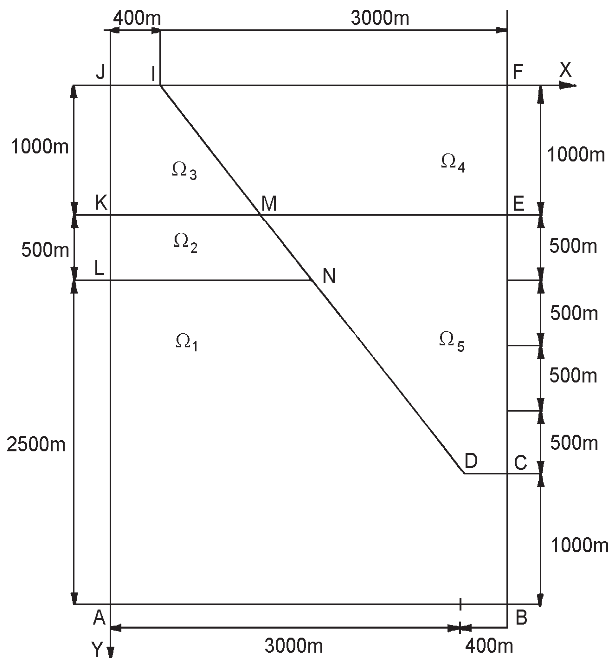

Figure 3. A half of the geometry of the geological region in 1951.

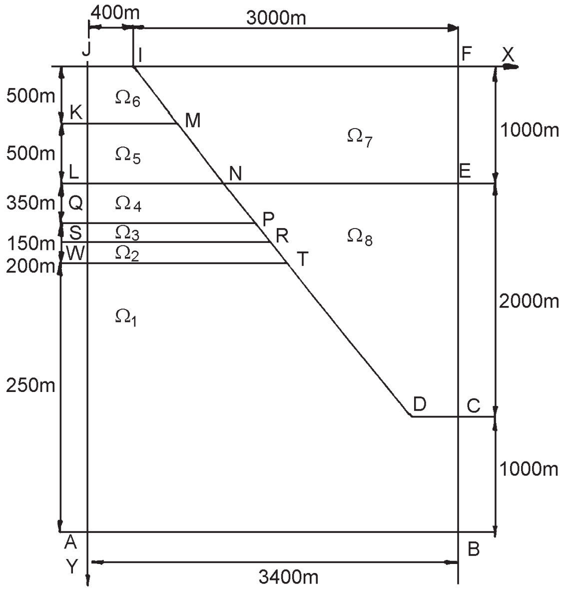

Figure 4. A half of the geometry of the geological region in 1994.

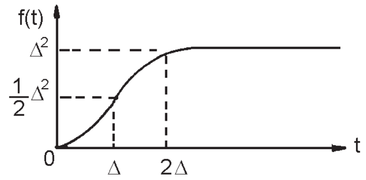

Figure 5. The time function of the dynamic source.

Two-dimensional in-plane transient wave propagation problem will be solved for the above geological columns. The plane strain state is considered. The soil material is homogeneous, isotropic and elastic. The displacement vector is with components $u_x(x, y, t)$ and $u_y(x, y, t)$.

The initial conditions are

$$u_i(x, y, t) = u_i^0(x, y) \text{ and } \dot{u}_i(x, y, t) = \dot{u}_i^0(x, y) \text{ for } t = t_0 \quad (11)$$

The boundary conditions are, see Figures (3) and (4).

It is convenient to represent the motion in the half-space as a superposition of the “free-field” motion and the waves scattered from the multi-layered geological column. The “free-field” motion consists of the plane incident wave and the reflected waves of the free boundary. The solution of the problem for transient elastic waves in a half-space without any layer due to the buried explosive load of type (10) is given and compared with other solutions in [4].

The boundary conditions (12) prescribe tractions on the part of the boundary S_p and displacements on the complementary part S_u, $S_B = S_p \cup S_u$, $S_p \cap S_u = \phi$. They are:

- On the surface of the half-space, at $y = 0$ (the boundary FJ) all tractions have to be zero

$$p_i(x, 0) = 0 \quad (12a)$$

- Due to the symmetry it is considered a half of the geometry and for $(x, y) \in JA$ the next boundary conditions are satisfied

$$P_y(x, y) = 0 \text{ and } u_x(x, y) = 0 \quad (12b)$$

- The influence of the geological column on the motion of the half-space has to vanish at sufficiently large distances, or Sommerfield boundary conditions have to be satisfied at infinity.

- On the boundary between two soil layers Ω_i and Ω_{i+1} inside the geological column the next continuity conditions have to be satisfied

$$u_i(x, y)\Big|_{S_{\Omega_i}} = u_i(x, y)\Big|_{S_{\Omega_i+1}} \quad (12c)$$

and the motion must be such that all the dynamic forces acting onto the boundary are in dynamic equilibrium

$$p_i(x, y)\Big|_{S_{\Omega_i}} = p_i(x, y)\Big|_{S_{\Omega_i+1}} \quad (12d)$$

- On the boundary between the geological column and the half-space the next boundary conditions have to be satisfied

$$u_i(x, y)\Big|_e = u_i^{free-field}(x, y)\Big|_e \quad (12e)$$

for $(x, y) \in AB, BC, CE, EF$, $e = \Omega_1, \Omega_4, \Omega_5$ for Figure (3) and $e = \Omega_1, \Omega_7, \Omega_8$ for Figure 4. The displacement field $u_i^{free\text{-}field}(x, y)$ is obtained after the solution of the problem for transient elastic waves in a half-space without any layer due to the buried explosive load of type (10).

The governing Eqs. (4), the initial conditions (11) and the boundary conditions (12) represent the considered boundary-value problem. The problem is solved following the numerical procedure described in detail in section 4.

Following the above-described numerical procedure, overcoming of weak and strong singularities in the obtained integrals and after satisfaction of the given boundary conditions, an algebraic complex system according to the unknowns is obtained.

The regular integrals are computed numerically employing the Gaussian quadrature scheme. The kernels of the type $\int_{\Gamma_k} P_{ij}^* d\Gamma_k$ have singularities like $O\left(\frac{1}{c \pm \xi}\right)$, for $c \in [-1, +1]$ that leads to the CPV integrals. The kernels of the type $\int_{\Gamma_k} U_{ij}^* d\Gamma_k$ have singularities like $O(ln(c \pm \xi))$ for $c \in [-1, +1]$, which leads to non-singular integrals. The singular integrals are solved analytically based on the asymptotic expansion for a small argument of the Bessel function. In all corner points, where the Helder continuity conditions are not satisfied, it is applied the Lachat and Watson concept [7] that $p_u^{s-1} = \sigma_{ij} n_j^{s-1} = p_u^s = \sigma_{ij} n_j^s$, for two different boundary elements Γ^{s-1} and Γ^{s}, which formed the corner.

6. Numerical Results

In this item the numerical results obtained after the solution of the boundary-value problem, formulated in section 5, using the numerical scheme which is described in section 4 based on the Direct Boundary Integral Equation Method combined with the finite difference procedure applied to the time variable are given. The advantages of this novel *BIEM* formulation are discussed in the above items.

The geometrical characteristics of the geological regions, presented in Figures (1) and (3) and Figures (2) and (4) are given in Tables (1) and (2). The mechanical properties of these geological columns are shown in Tables (1) and (2), see part I of the paper.

The depth of the applied dynamical load is 4500*m* and its location is (0.0, 4500*m*). The epicentre distance of the receiver is x = 1400*m*. The time step at the realization of the numerical algorithm is 0.001*s* and the value of the parameter θ is 1.4.

The theoretical seismograms in the time domain for the horizontal u_x and vertical u_y components of displacement at the receiver, located at the free surface of the layered half-space at distance x = 1400*m*, are obtained. They are perturbed by a buried explosive seismic source (that is a line source of dilatation acting at x_S =(0.0,4500)). The time function of the source $f(t)$, see Eq. (10) is assumed to be parabolic ramp function which half-rise time Δ is taken as Δ=0.1s.

Figures (6a)-(6b) show the horizontal and vertical component of displacement in the time domain at a point (1400,0.0), when the boundary-value problem is solved for the geological region given in Figure (1).

The theoretical seismograms in the time domain for the horizontal and vertical displacements obtained at a point (1400,0.0) when the boundary-value problem is solved for the geological region given in Figure (2) are shown in Figures (7a)-(7b).

One can see that the transient dynamic responses of one and the same real geological region - a multi-layered

Table 1. Geometrical parameters of the geological column N1.

Boundary	Length [m]
AB	3400
BC	1000
CE	2000
EF	1000
FI	3000
IJ	400
JK	1000
KL	500
LA	2500

Table 2. Geometrical parameters of the geological column N2.

Boundary	Length [m]
AB	3400
BC	1000
CE	2000
EF	1000
FI	3000
IJ	400
JK	500
KL	500
LQ	350
QS	150
SW	200
WA	2300

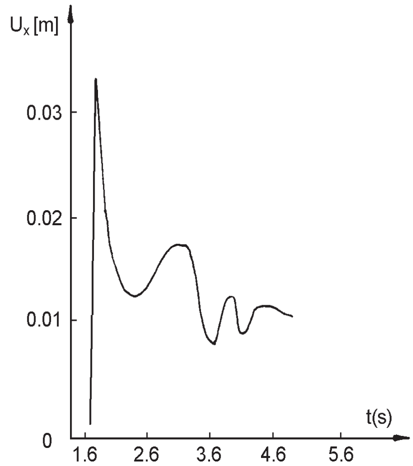

Figure 6a. Theoretical seismogram for the horizontal displacement in column N1.

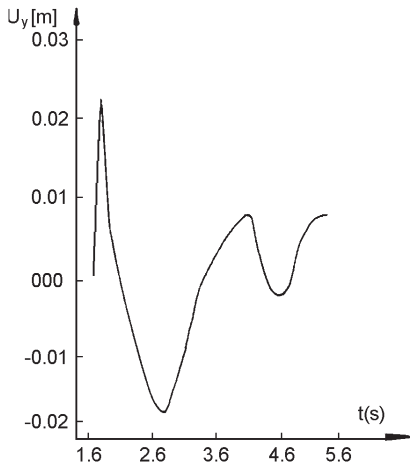

Figure 6b. Theoretical seismogram for the vertical displacement in column N1.

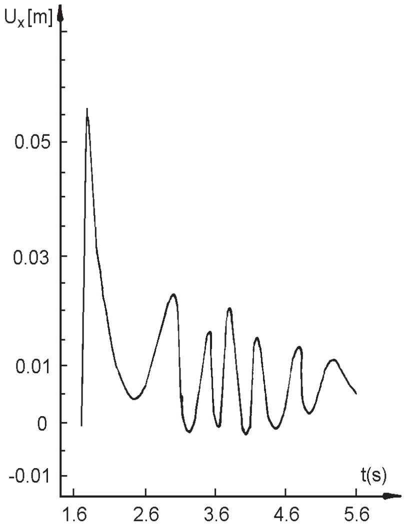

Figure 7a. Theoretical seismogram for the horizontal displacement in column N2.

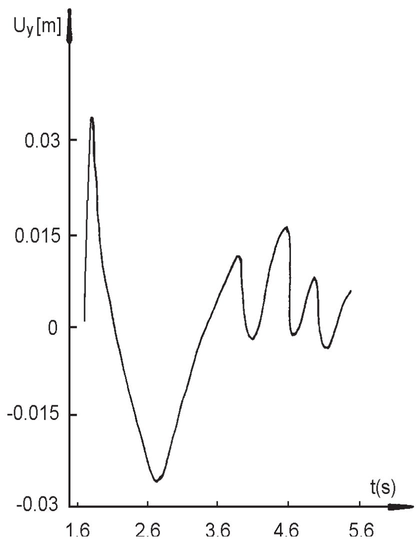

Figure 7b. Theoretical seismogram for the vertical displacement in column N2.

soil media with existence of salt ore deposits, obtained at different exploitation periods (in 1951 and in 1994) are different. The change of the geological situation during the years due to the exploitation process in the salt ore deposits leads to the different transient responses obtained at the application of a buried dynamic transient load.

7. Conclusion

An application of the novel two-dimensional formulation of the direct boundary integral equation method proposed in [4] has been applied here for multi-layered geological columns with complex geometry. This application concerns two real geological situations for multi-layered soil media with existence of salt ore deposits. Figures (1) and (2) show one and the same geological region but in different periods of time. One can see that the change of the geological situation during the years when the exploitation of the salt ore deposits has been done leads to the change of the transient wave picture due to the buried dynamic load. The time records of the surface responses are computed by the proposed in [4] novel formulation of *DBIEM* for the real geological situations and different wave fields are obtained. These results show that the changes in the soil region lead to the change in its dynamic response, i.e. to the change in the obtained theoretical seismograms. All this assures us that the exploitation process leads to the changes in the geological situation of the region, respectively to the changes of the soil response during eventual earthquake.

Finally we would like to underline once again that the present hybrid formulation, combined the finite difference procedure with the direct boundary integral equation method employs the fundamental solution depending neither on the frequency nor on the time variable. This is a serious advantage of the proposed method. This work shows that the method proposed here works well even in the cases of multi-layered geological regions with complex geometry.*

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Appendix

The function $U^*_{kj}(x, y, x_0, y_0, \omega)$ is the fundamental solution of the system of Eq. (15) in Part I of this paper and the function $P^*_{kj}(x, y, x_0, \; y_0, \omega)$ is the corresponding traction

$$U^*_{kj}\left(x^p - x^q, y^p - y^q, \omega\right) = \frac{1}{2\pi\mu}\left[\psi\delta_{kj} - \chi r_{,k} r_{,j}\right]$$

$$P^*_{kj}\left(x^p - x^q, y^p - y^q, \omega\right) = \frac{1}{2\pi}\left\{\left(\frac{\partial\psi}{\partial r} - \frac{\chi}{r}\right)\left(\delta_{kj}\frac{\partial r}{\partial n} + r_{,k} n_j\right)\right.$$

$$\left. - 2\frac{\chi}{r}\left(r_{,j} n_k - 2 r_{,k} r_{,j}\frac{\partial r}{\partial n}\right)\right\} - \frac{1}{2\pi}\left\{2\frac{\partial\chi}{\partial r} r_{,k} r_{,j}\frac{\partial r}{\partial n} - \left(\frac{V_P^2}{V_S^2} - 2\right)\right.$$

$$\left. \times\left(\frac{\partial\psi}{\partial r} - \frac{\partial\chi}{\partial r} - \frac{\chi}{r}\right) r_{,j} n_k\right\}$$

where (x^p, y^p) and (x^q, y^q) are the field point and the running point respectively

$$r = \sqrt{(x^p - x^q)^2 + (y^p - y^q)^2}$$

$$r_{,x^q} = \frac{r_x}{r} = \frac{x^q - x^p}{r}$$

$$r_{,y^q} = \frac{r_y}{r} = \frac{y^q - y^p}{r}$$

$$\frac{\partial r}{\partial n} = r_{,x^q} n_{x0} + r_{,y^q} n_{y0}, \quad s = -i\omega$$

$$\chi = K_2\left(\frac{sr}{V_S}\right) - \frac{V_S^2}{V_P^2} K_2\left(\frac{sr}{V_P}\right)$$

$$\psi = \frac{1}{2}\left[K_2\left(\frac{sr}{V_S}\right) - \frac{V_S^2}{V_P^2} K_2\left(\frac{sr}{V_P}\right)\right] + \frac{1}{2}\left[K_0\left(\frac{sr}{V_S}\right) + \frac{V_S^2}{V_P^2} K_0\left(\frac{sr}{V_P}\right)\right]$$

The function $K_m(z) = -\frac{1}{2} i\pi(i)^m H_m^{(2)}(iz)$ for $z = -\frac{i\omega}{V_S}$ or $z = -\frac{i\omega}{V_P}$ is represented with the modified Bessel functions of the second type.

The asymptotic representations of the functions U^*_{kj} and P^*_{kj} for $r \to 0$ are

$$\left(U^*_{kj}\right)^{as} \approx -\frac{1}{4\pi\mu}\left\{\left[\left(1 + \frac{V_S^2}{V_P^2}\right) lnr + ln\frac{\omega}{2V_S} + \frac{V_S^2}{V_P^2} ln\frac{\omega}{2V_P}\right]\delta_{kj} - A\right\}$$

$$\left(P^*_{kj}\right)^{as} \approx -\frac{1}{2\pi r}\frac{V_S^2}{V_P^2}\left\{\left[\delta_{kj} + 2\left(1 + \frac{V_S^2}{V_P^2}\right) r_{,k} r_{,j}\right]\frac{\partial r}{\partial n} - BB\right\}$$

where

$$A = -\frac{1}{4\pi\mu}\left\{\left(1 - \frac{V_S^2}{V_P^2}\right) r_{,k} r_{,j} + i\frac{3\pi}{2}\left(1 + \frac{V_S^2}{V_P^2}\right)\right\} \text{ and}$$

$$BB = -\frac{1}{2\pi r}\frac{V_S^2}{V_P^2}\left\lfloor n_k r_{,j} - n_j r_{,k}\right\rfloor.$$