JSEE Vol. 17, No. 4, 2015

This is an extended version of the paper presented in SEE7 conference, peer-reviewed again and approved by the JSEE editorial board.

Reliability Analysis of Liquefaction Utilizing Monte Carlo Simulation Based on Simplified Stress Method

Jalal Kasebzadeh[1], Ali Noorzad[2*], and Ahmad Reza Mahboubi[3]

1. M.Sc. Graduate of Geotechnical Engineering, Shahid Beheshti University, Tehran, Iran
2. Assistant Professor, Faculty of Civil, Water and Environmental Engineering, Shahid Beheshti University, Tehran, Iran, * Corresponding Author; email: a_noorzad@sbu.ac.ir
2. Associate Professor, Faculty of Civil, Water and Environmental Engineering, Shahid Beheshti University, Tehran, Iran

Received: 31/08/2015
Accepted: 16/12/2015

ABSTRACT

Liquefaction analysis is one of the most challenging issues in seismic geotechnical engineering. The unknown factors and pertinent uncertainties involved in the evaluation of liquefaction potential make the problem to be complicated. Liquefaction evaluation include deterministic and probabilistic methods. Deterministic methods are simple but they are not capable to consider the uncertainties. With regard to heterogeneous nature of the soil and probabilistic nature of earthquake loading, it seems that deterministic method is not sufficient for evaluation of liquefaction. Reliability methods are able to capture the uncertainties depending on variability of soil parameters and also to determine the factor of safety proportional to the acceptable risk. In recent years, reliability analysis of liquefaction has been done using approximated method. In the present research, reliability analysis of liquefaction triggering has been discussed using Monte Carlo simulation that is an accurate method. For this purpose, the parameters earthquake magnitude (M_w*), maximum horizontal acceleration (*a_{max}/g*), total stress (*σ_v*), effective stress (*σ'_v*), fines content percent (FC), and SPT blow count (*N_{SPT}*) are selected as stochastic parameters and the probability of liquefaction has been estimated. Application of the proposed method to the 233 well-documented case studies verify that deterministic method is not accurate enough to predict the liquefaction and reliability analysis should be used instead. Besides, the sensitivity tests indicate that the SPT blow count is the most influential parameter in liquefaction evaluation and large number of iterations is not required in Monte Carlo Simulation and the results converge after a specific number of iterations.*

Keywords:
Liquefaction; Monte Carlo simulation; Geotechnical uncertainty; Reliability analysis; Probability of liquefaction

1. Introduction

The requirements of reliable design of structures subjected to strong ground shaking have attracted the attention of many researchers. A designer should consider pertinent problems of liquefaction in view of the safety of the structures. The soil liquefaction phenomenon is an important issue of concern to earthquake geotechnical engineers in recent years. The liquefaction phenomenon happens when saturated granular medium loses its shear strength due to the increase in pore water pressure under seismic loading. With the occurrence of this phenomenon, saturated sandy soils will lose their strength and soil particles will flow. According to the case studies, soil liquefaction is one of the most important reasons of damages to lifelines, buildings and infrastructures during earthquakes. Liquefaction can cause large displacements in the ground, soil failures, reduction of bearing capacity, differential settlements in foundations, and sand boiling. This phenomenon has been observed in many earthquakes such as Alaska (1964), Niigata (1964), Loma Prieta (1989), Kobe (1995), Chi-Chi (1999) and recently at Bushehr, Iran (2013). The expenses of soil improvement techniques are usually very high, hence accurate evaluation of soil liquefaction potential can reduce the overall cost of the projects in addition to guarantee the safety of the project.

Selection of geotechnical parameters is always one of the challenging issues to geotechnical engineers due to the uncertainties of soil and rock. Engineers usually apply large safety factors to consider the uncertainties. Although, this approach makes the projects to be a rather expensive but not reliable enough because there is no explicit relation between factor of safety and probability of failure, and it makes the engineering judgment to be complicated [1]. In recent years, probabilistic methods have been extended to overcome this deficiency.

In general, it is possible to categorize the geotechnical uncertainties into two groups: inherent uncertainties and epistemic uncertainties [2]. Soil inherent uncertainties are due to the nature of variability of soil parameters in different locations and time. With regard to the nature of these uncertainties, the effect of them should not be disregarded. The second category of uncertainties is because of the lack of information and knowledge in geotechnical engineering. Epistemic uncertainties include the measurement errors, statistical uncertainties, and model uncertainties [3]. Uncertainties related to design parameters obtained from laboratory data are among the second category. Measurement errors could be due to apparatus or user errors [4]. Model uncertainty is also the result of idealization of physical models [5]. It is possible to reduce the epistemic uncertainties by increasing the number of samples for testing and observations. Considering the uncertainties in geotechnical engineering, it is not recommended to take soil parameters with deterministic values. On the other hand, the appropriate method is to use probabilistic approach to find soil parameters for design purposes. Reliability methods that are based on probabilistic approach provide the opportunity to quantify the uncertainties and can be used as a supplementary tool for deterministic method [6]. Reliability methods are able to capture the uncertainties depending on the variability of soil parameters and to determine the factor of safety proportional to the acceptable risk.

This paper deals with the probability of liquefaction under dynamic loadings depending on the variability of affecting parameters utilizing probabilistic analysis. The advantages and disadvantages of the deterministic and probabilistic analysis of liquefaction potential are also discussed.

2. Deterministic Evaluation of Liquefaction Triggering

The most conventional procedure for the liquefaction evaluation is simplified stress method. This method has suggested by Seed and Idriss in 1971 and has been reviewed several times [7]. Although there have been considerable advances in understanding of seismic ground motion and liquefaction phenomenon, most of the researchers involved made minor modifications, and the simplified procedure preserved its main structure. However, simplified stress procedure has been extended based on in-situ tests results such as Standard Penetration Test [8-16], Cone Penetration Test [12] and [17-22] and Shear Wave Velocity [12, 23]. These methods compare CSR (Cyclic Stress Ratio) and CRR (Cyclic Resistance Ratio) and calculate the factor of safety against liquefaction potential [24]. In the present research, the most recent simplified method, which has been developed by Idriss and Boulanger [16], is used. The CSR is given by Eq. (1):

$$CSR = \left(\frac{a_{max}}{g}\right)\left(\frac{\sigma_{v0}}{\sigma'_{v0}}\right)\left(r_d\right) \tag{1}$$

where a_{max} is the maximum horizontal acceleration, g is the acceleration of gravity, σ_v is the total stress, σ'_v is the effective stress and r_d is the shear stress reduction factor.

Cyclic Resistance Ratio (CRR) is usually correlated to an in-situ parameter such as Standard Penetration Test (SPT) blow counts (N), Cone Penetration Test (CPT) resistance and Shear Wave Velocity. Idriss and Boulanger [16] have used SPT results and proposed Eq. (2) to calculate the CRR.

$$CRR_{M=7.5,\sigma'_{v0}=1atm} = exp\left[\left(\frac{(N_1)_{60cs}}{14.1}\right)+\left(\frac{(N_1)_{60cs}}{126}\right)^2 - \left(\frac{(N_1)_{60cs}}{23.6}\right)^3+\left(\frac{(N_1)_{60cs}}{25.4}\right)^4 - 2.8\right] \tag{2}$$

in which $(N_1)_{60cs}N$ is the modified SPT blow counts.

Finally, the factor of safety against liquefaction is determined by the ratio of CRR to CSR as given in Eq. (3). It is obvious that if $FS < 1$, liquefaction occurs, and for the case of $FS > 1$, it is safe.

$$FS = \frac{CRR_{M_w,\sigma'_{v0}}}{CSR} \tag{3}$$

For a comprehensive explanation of the method, the interested readers may refer to Idriss and Boulanger paper [16].

3. Reliability Analysis Methods in Liquefaction Analysis

Reliability analysis methods are usually divided into three categories: analytical, approximate and simulation methods. In analytical methods, probability density functions of input parameters are expressed mathematically. Then, the equation of performance function (e.g. safety factor) is integrated over input variables. Accordingly, probability density function of the performance function will be determined. These methods are limited to problems with few numbers of stochastic variables. Fewer studies have been done on this subject due to their mathematical complexity. Jointly Distribution Random Variables (JDRV) method is one of the mathematical techniques of this category [25-28]. JDRV method uses numerical integration technique to find probability distribution function (PDF) of performance function, but it is not able to consider correlation coefficient between stochastic parameters.

Approximated methods compute the probability of events with some indicators such as mean value and variance of input stochastic parameters. Common approximation methods are based on three methods: First Order Reliability Method (FORM) [29], First Order Second Moment (FOSM) [30], and Point Estimation Method (PEM) [31]. Each of these methods utilizes several simplifying assumptions for prediction of failure probability that somewhat reduces the accuracy. Approximate methods are able to estimate the mean and variance of performance function, but they do not provide any information concerning the shape of its PDF. Hence, the probability of events is determined just based on the assumed PDF for performance function (usually using normal distribution function). In recent years, most reliability analyses of liquefaction have been done using these methods [32-42].

Simulation methods are among the accurate reliability methods. These methods predict the probability of event by simulating stochastic input parameters and implementing in repetitive calculations. In mathematics, these methods have been used for complex problems the closed-form solution of which is not possible (i.e. large degree of integration) [43]. Nowadays, regarding too rapid development of computer technology and available personal computer utilization of these methods has been increased in engineering problems. Monte Carlo simulation method is one of the most applicable methods of this category. This method has been discussed in the next section.

4. Monte Carlo Simulation Method

Monte Carlo Simulation (MCS) is a numerical process of repeatedly calculating a mathematical or empirical operator F(X) in which the variable $X = [x_1; x_2; \ldots ; x_n]$ within the operators are random or contain uncertainty with prescribed probability distributions [43-44]. MCS is an accurate reliability analysis method and is applicable for any limit state approach [45-46]. It has been widely used in reliability analysis of geotechnical engineering problems such as slope stability, retaining walls, foundations and risk assessment of complex engineering problems. In the analysis process, stochastic values for each of the input parameter are selected based on its statistical parameters. The probability density function of stochastic parameters can have any shape but normal, log-normal and beta distribution functions are usually used based on the characteristics of the stochastic variables. These values are used to calculate the performance function. This procedure is repeated for many times to obtain proper statistical distribution for performance function. Statistical analysis of this distribution enables the user to calculate the mean and standard deviation of performance function and finally predict the probability of events. Generally, this method consists of four steps as follows [46]:

1. Generating stochastic values for each of stochastic variables according to assigned probability density function.
2. Computing performance function using a proper deterministic method based on generated values in previous step.
3. Repeating steps 1 and 2 for as many times as required.
4. Determining probability distribution function of performance function and calculate the probability of events.

5. Evaluation of Liquefaction Triggering Utilizing Monte Carlo Simulation

Due to the probabilistic nature of earthquakes, it is required to analyze the response of earthquake loading by probabilistic methods. It is possible to divide the uncertainties of liquefaction into two categories: parameter uncertainties and model uncertainties. In the present research, the reliability analysis of liquefaction potential considering parameter uncertainties has been investigated. For this purpose, the factor of safety function is selected as performance function, and the parameters earthquake magnitude (M_w), maximum horizontal acceleration (a_{max}/g), total stress (σ_v), effective stress(σ'_v), fines content percent (FC) and *SPT* blow count (N_{SPT}) are chosen as stochastic variables according to the Tables (1) and (2). These values are chosen based on previous researches [46]. With respect to Eq. (2), shear stress reduction factor, r_d, is a function of M_w and is also a variable.

There have been a few researches concerning the selection of a proper probability distribution function for parameters. Most of the previous studies have suggested using Normal and log-Normal distribution function for geotechnical purposes [46]. In the present research, it is decided to choose Normal distribution function for the parameters that their coefficient of variations are less than 25 percent and log-Normal or Beta distribution function for the parameters that their coefficient of variations are more than 25 percent.

Table 1. Stochastic variables characteristics.

Parameter	Probability Density Function	Coefficient of Variation
N_{SPT}	Normal	20
a_{max}	Normal	5
M_w	Normal	5
σ_v	Normal	15
σ'_v	Normal	15
FC	Normal	10

Table 2. Correlation coefficient between variables.

Parameter	N_{SPT}	a_{max}	M_w	σ_{v0}	σ'_{v0}	FC
N_{SPT}	1	0	0	0	0	0
a_{max}	0	1	0.9	0	0	0
M_w	0	0.9	1	0	0	0
σ_{v0}	0	0	0	1	0.9	0
σ'_{v0}	0	0	0	0.9	1	0
FC	0	0	0	0	0	1

Stochastic variables are generated based on mean, coefficient of variation, correlation coefficient and probability distribution function. Each set of the generated stochastic values is put into the performance function to calculate the factor of safety for each trial. This procedure is repeated for numerous times to find the probability density function of the performance function. The probability of liquefaction (PL) is equal to the area under the curve of probability density function with safety factors less than 1. It is also possible to calculate it using Eq. (4). Different classes of probability of liquefaction (PL) are given in Table (3) [36].

$$P_L = \frac{N_L}{N} \tag{4}$$

in which P_L is the probability of liquefaction, N_L is the number of trials with factor of safety less than 1 and N is the total number of Monte Carlo trials.

Table 3. Classification of probability of liquefaction [37].

Class	Probability (%)	Description
1	P_L<15	Almost certain that it will not liquefy
2	15 < P_L<35	Liquefaction unlikely
3	35 < P_L<65	Liquefaction and non-liquefaction equally likely
2	65 < P_L<85	Liquefaction very likely
5	85 < P_L<100	Almost certain that it will liquefy

5.1. A Typical Case as an Example

To demonstrate the efficiency and accuracy of Monte Carlo simulation in prediction of liquefaction, a typical example has been considered. The characteristics of the example are selected from case printed in literature [16]. The deterministic parameters of this example are given in Table (4) and stochastic parameters are selected based on Tables (1) and (2). In the proposed method, a wide range of stochastic parameters are selected instead of choosing a deterministic value.

Table 4. Deterministic parameters of typical example [16].

Z (m)	**M_W**	$\left(\frac{a_{max}}{g}\right)$	$\sigma_v\,(\frac{kN}{m^2})$	$\sigma'_v\,(\frac{kN}{m^2})$	**N_{SPT}**	**FC (%)**
7.5	7.0	0.35	141	104	17.3	4

In order to do the Monte Carlo simulation of liquefaction triggering, 1.2 million iterations have been accomplished and probability density function of performance function has been determined and is shown in Figure (1). The probability of liquefaction is displayed in blue region in Figure 1. In this example, the probability of liquefaction is equal to 60.5 percent. According to the Table (3), Liquefaction and non-liquefaction are equally likely in this case. Estimating the probability of liquefaction provides the opportunity to judge proportional to acceptable risk, and it will facilitate engineering judgment in this issue.

In approximated reliability methods usually assumes that the probability distribution of factor of safety has normal distribution function [6]. Regarding to Figure (2), it is obvious that it follows

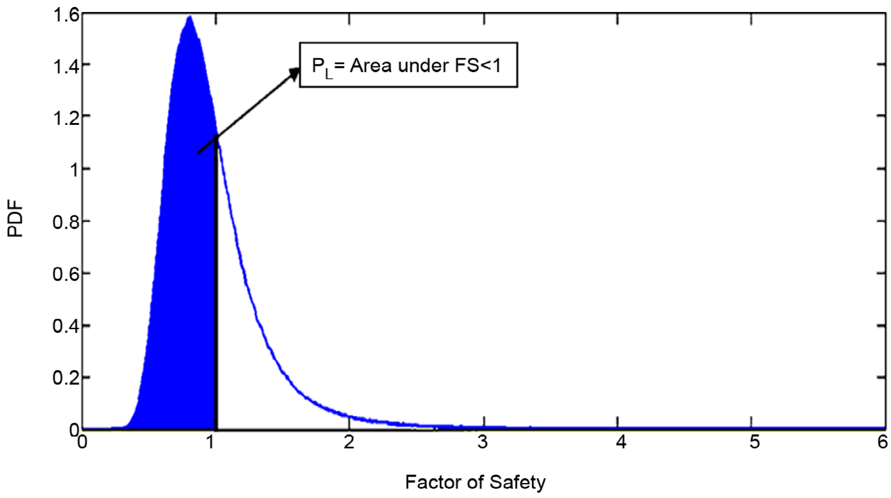

Figure 1. Probability density function (PDF) of factor of safety.

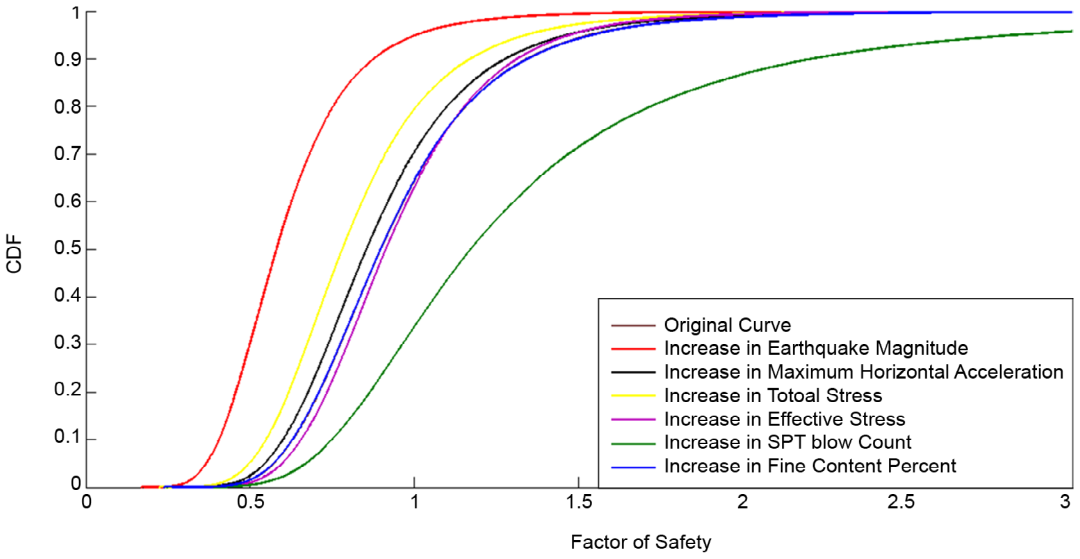

Figure 2. Sensitivity analysis of the proposed model to input parameters.

Log-normal distribution. Therefore, it is suggested to use Log-Normal distribution for calculating the probability of liquefaction in approximated methods.

5.2. Sensitivity Analysis

5.2.1. Sensitivity of Input Parameters

In order to analyze the response of the proposed model with respect to changes in input parameters and to determine the most affecting parameter in the evaluation of liquefaction, a sensitivity analysis has been done. For this purpose, the amount of each input parameters increased equal to one standard deviation and the probability of liquefaction is calculated. The results of this sensitivity analysis are given in Figure (2). It is shown that with the increase of parameters N_{SPT}, FC and σ'_v, the cumulative distribution of factor of safety shifts rightward, which indicate that, the probability of liquefaction has been decreased. With the increase in the parameters M_W, a_{max} and σ_v, the cumulative distribution of factor of safety shifts leftward indicating that the probability of liquefaction has been increased. The amounts of the changes of the probability of liquefaction are shown in Table (5). The results show that, the SPT blow count is the most effective parameter and the percentage of fines content (FC) have less effect on the evaluation of liquefaction potential

5.2.2. Effect of Correlation Coefficient

Two variables may be related to one another, as indicated by a scatter plot. The correlation coefficient is a measure of the degree of linear dependence between two variables. The sample correlation coefficient, ρ is given as follows:

$$\rho_{xy} = \frac{\sum_{i=1}^{n}\left[(x_i-\mu_x)(y_i-\mu_y)\right]}{\sqrt{\sum_{i=1}^{n}(x_i-\mu_x)^2\sum_{i=1}^{n}(y_i-\mu_y)^2}} \tag{5}$$

where x_i and y_i are paired observations of the two variables, and μ_x and μ_y are mean value of x and y parameters, respectively. The sample correlation coefficient ranges between -1.0 and 1.0. A value of zero for ρ indicates no linear dependence between the two variables. A negative value of ρ indicates that one variable tends to decrease as the other increases, while a positive value indicates that one variable tends to increase as the other variable increases. The closer the absolute value of ρ is to 1.0, the stronger the linear relationship between the two variables [47].

As mentioned in section 4, stochastic number generation plays an important role in Monte Carlo simulation method. If the stochastic values do not have appropriate correlation, generated stochastic values will affect the results. Generated stochastic values without/with considering correlation coefficient have shown in Figures (3) and (4), respectively. Comparison of these figures demonstrate that considering correlation coefficient is necessary for Monte Carlo simulation. Figure (3) shows that stochastic values do not follow a proper trend. For instance, the effective stress will exceed the total stress in some cases. Accordingly, correlation coefficients between stochastic parameters should be taken in MCS method.

5.2.3. Effect of Number of Iterations

The number of required Monte Carlo trials is dependent on the level of confidence in the solution and the amount of stochastic variables. Based on the statistical theory, Eq. (6) has been recommended for the number of iterations [46]:

$$N = \left(\frac{d^2}{4(1-\varepsilon)^2}\right)^m \tag{6}$$

where N is the number of Monte Carlo trials, d is the normal standard deviation corresponding to the level of confidence, ε is the desired level of confidence and m is the amount of stochastic variables. Table (6) represents some confidence level (ε) with corresponding standard deviation (Std).

With regard to the number of stochastic variables

Table 5. The amount of changes in the probability of liquefaction corresponding to increase of parameters.

Parameter	Shift in N_{SPT}	Shift in M_W	Shift in a_{max}	Shift in σ_v	Shift in σ'_v	Shift in FC
Change (%)	-28.55	11.22	5.74	14.23	-5.51	-0.06

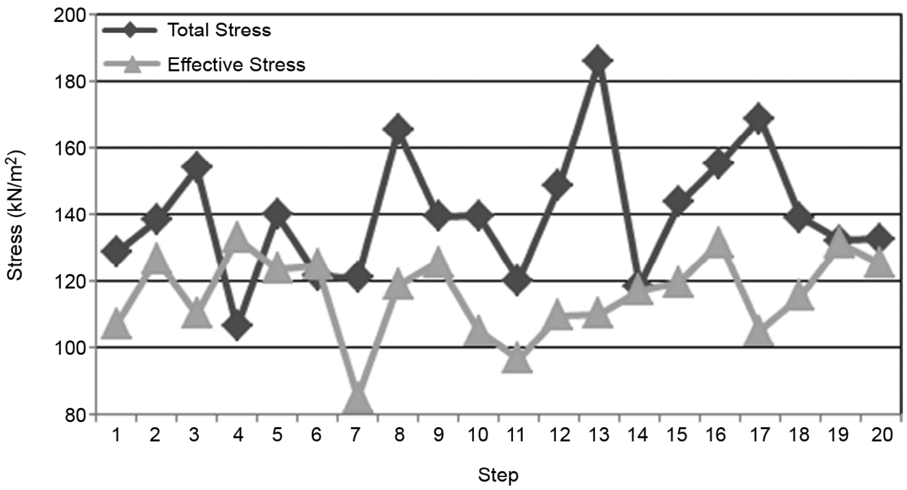

Figure 3. Generated stochastic values without applying correlation coefficients.

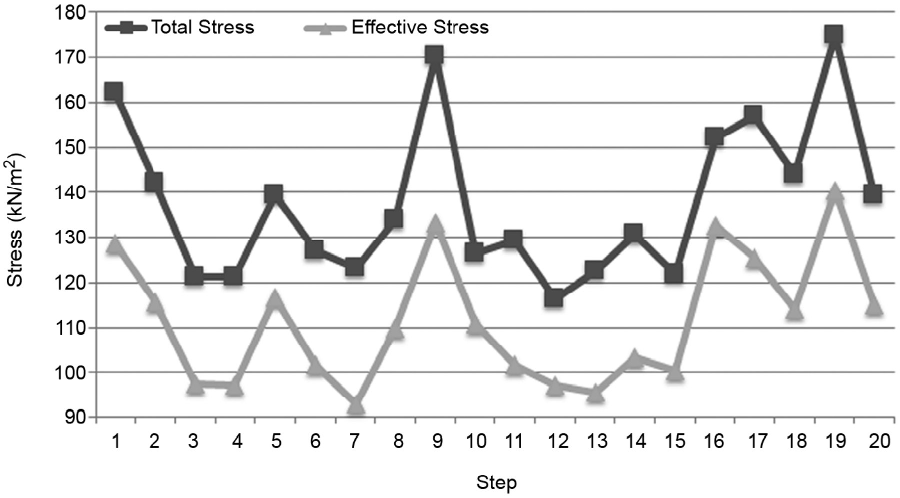

Figure 4. Generated stochastic values when correlation coefficient applied.

Table 6. Standard deviations according to confidence levels.

Confidence Level (ε)	Standard Deviation (Std)
80%	1.282
90%	1.645
95%	1.960
99%	2.576

and confidence level, 1.2 million iterations have been done, but it seems that there is no need to do such a large trial. In order to investigate the effect of number of iterations on the results, a sensitivity analysis has been examined and the results are shown in Figure (5). It demonstrates that the probability of liquefaction converges after about 500,000 trials and therefore a large number of iterations is not required.

6. Verification of the Proposed Methodology

To verify the application of Monte Carlo simulation method in liquefaction analysis, 233 well-documented case studies of liquefaction that was reported in literature has been examined. In this section, a case study, namely Loma Prieta earthquake with a magnitude of 7 Richter at San Francisco in October 1989, has been discussed in detail. During this strong ground shaking, major failures occurred in vulnerable sites to liquefaction phenomenon. Borehole specifications in different sites of that zone have been reported by Idriss and Boulanger [16] and other researchers. Some parts of these data are presented in Table (6). It also gives the factor of safety with deterministic approach and liquefaction probability is estimated by MCS. Comparison between the results of deterministic analysis and actual occurrence of liquefaction in this case study indicates that the deterministic analysis method is not reliable enough to predict the event occurred. For example in site numbers 13, 18 and 23, despite the fact that the factor of safety is more than 1, liquefaction had occurred. In site numbers 9, 11 and 15, the factor of safety is less than 1, but there was no evidence of liquefaction occurrence. On the other hand, the results of MCS are in good agreement with the actual occurrence

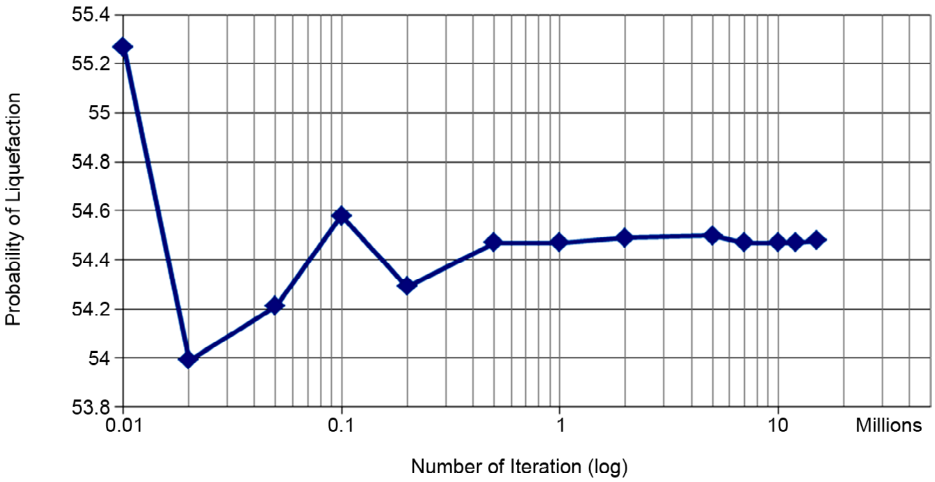

Figure 5. Effect of number of iterations on the Monte Carlo simulation results.

Table 7. Liquefaction probability estimation utilizing Monte Carlo Method -Loma Prieta earthquake (1989).

No.	Site	M_W	a_{max}/g	Z	σ_{v0}^{*}	$\sigma'_{v0}{}^{*}$	N_{SPT}	FC%	Liquefy?	FS	PL%
1	Alameda Bay	6.93	0.24	6.5	125	91	37	7	No	10.6	0
2	Faris Farm	6.93	0.37	6	106	92	9	8	Yes	0.56	99.44
3	General Fish	6.93	0.28	2.5	45	35	16.9	5	No	1.37	13.5
4	Hall Avenue	6.93	0.14	4.6	75	64	4.6	30	No	1.56	0.09
5	Marine Laboratory b1	6.93	0.28	4.6	87	65	11	3	Yes	0.77	84.19
6	Marine Laboratory b2	6.93	0.28	3.5	65	55	13	3	Yes	0.92	54.45
7	Marine Lab UBC-6-12	6.93	0.28	5.3	102	64	12	3	Yes	0.85	68.47
8	Marine No. 3 EB-1	6.93	0.28	2	35	35	18	1	No	1.96	1.5
9	Marine No. 3 EB-5	6.93	0.28	3.4	63	47	12	1	No	0.83	68.55
10	Mbari No. 4	6.93	0.28	3.4	62	48	18	5	No	1.31	14.5
11	Mbari Technology	6.93	0.28	3.4	62	48	12	4	No	0.86	66.35
12	Miler Farm CMF3	6.93	0.39	6.2	114	101	9.2	32	Yes	0.69	94.9
13	Miler Farm CMF5	6.93	0.39	7	130	108	20	13	Yes	1.03	47.9
14	Miler Farm CMF8	6.93	0.39	6	111	95	8.8	25	Yes	0.66	95.85
15	Miler Farm CMF10	6.93	0.39	8.4	158	105	19	20	No	0.93	55.9
16	Poo 7-2	6.93	0.28	6.3	121	89	14.4	3	Yes	0.8	77.8
17	Poo 7-3	6.93	0.28	6.3	121	89	16	3	Marginal	0.88	66.8
18	Por 2 & 3 & 4	6.93	0.18	5.9	97	73	4.3	50	Yes	1.07	27.8
19	Sandholtt UC-B10	6.93	0.28	3	55	43	9.5	2	Yes	0.89	60.5
20	Sandholtt UC-B10	6.93	0.28	6.1	115	73	26	5	No	6.83	1.87
21	SFOBB-1 & 2	6.93	0.27	6.3	118	86	7.5	8	Yes	0.6	98.75
22	State Beach UC-B1	6.93	0.28	3.4	61	46	6.3	1	Yes	0.64	97.2
23	State Beach UC-B2	6.93	0.28	4.9	90	67	12.8	1	Yes	1.04	38.38
24	Treasure Island	6.93	0.16	6.5	116	67	4.3	20	Yes	0.9474	51.9
25	Wood Marine UC_B4	6.93	0.28	1.8	32	25	6.7	35	Yes	0.85	65.2

*Unit=kN/m^2

evidences and the mentioned criteria in Table (3). For example in site numbers, 1, 3, 4, 8, 10 and 20, in which the probability of liquefaction are less than 15 percent, there was no evidence of liquefaction occurrence. Besides, in site numbers, 2, 12, 14, 21 and 22, in which the probability of liquefaction determined more than 85 percent, the liquefaction had occurred. Moreover, the application of MCS to other case studies show a good agreement to the Loma Prieta cases that are given in Appendix A. Regarding these results, again it confirms that the proposed method has a better estimation in comparison to the deterministic method. The probabilistic method also provides this opportunity to have a good judgment.

7. Conclusions

Liquefaction potential is a probabilistic phenomenon due to the uncertain nature of earthquake and variability in soil deposits. Regarding to pertinent uncertainties, it seems that deterministic method is not suitable for liquefaction evaluation. On the other hand, the factor of safety criterion makes engineering judgment to be complicated because there is no explicit relation between the factor of safety and probability of liquefaction. Reliability methods, which consider uncertainties and estimate the probability of liquefaction can facilitate the engineering judgment. In the present research, the reliability analysis of liquefaction triggering utilizing Monte Carlo simulation has been examined. Based on the results presented in this paper, the following conclusions are drawn.

- The results obtained from Monte Carlo simulation method and deterministic method indicate that the deterministic method is not reliable enough in comparison to the probabilistic results, which are in good agreement with the real liquefaction events. Thus, this procedure is recommended to be used for other susceptible sites to liquefaction potential.
- The sensitivity analysis shows that the SPT blow count is the most influential parameter and the percentage of fines content have less effect on the analysis.
- Considering the correlation coefficient is necessary for MCS and should be considered in the analysis.
- In approximated methods, it is usually assumed that the probability density function of factor of safety follows normal distribution; however, in this research, it is shown that log-normal distribution gives a better prediction. It is suggested to use log-normal distribution for calculating the probability of liquefaction in approximated methods.
- Based on the sensitivity analysis, it is shown that there is no need to use a large number of trials for Monte Carlo simulation.

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Appendix A. Application of proposed method in 233 well-documented case studies.

Site	M_w	a_{max}/g	Z	σ_{v0}^{*}	$\sigma'_{v0}{}^{*}$	N_{SPT}	FC	Liq	FS	PL
1944, M=8.1 Tahnakai earthquake-Dec 7										
Komei	8.1	0.2	5.2	98	68	5.9	10	Yes	0.6	99.4
Ienaga	8.1	0.2	4.3	80	61	2.3	30	Yes	0.6	99.1
Meiko	8.1	0.2	3.7	69	39	1	27	Yes	0.4	100.0
1948, M=7.3 Fukui earthquake- June 28										
Shoneji Temple	7	0.4	4	75	48	8	0	Yes	0.4	100.0
Takaya 45	7	0.35	7.5	141	104	17.3	4	Yes	0.9	63.3
1964, M=7.6 Niigata earthquake- June 16										
Arayamotomachi	7.6	0.09	3.3	63	41	2.6	5	Yes	1.0	33.2
Cc17-1	7.6	0.16	7	132	72	8	2	Yes	0.7	94.1
Cc17-2	7.6	0.16	5.3	85	43	7.9	8	Yes	0.8	84.7
Kagawishi-cho	7.6	0.162	3.8	71	53	4.5	5	Yes	0.7	89.6
old Town- 1	7.6	0.18	7	132	81	18	2	No	1.4	12.7
old Town- 2	7.6	0.18	10.1	190	109	20	2	No	1.3	21.0
Rail Road-1	7.6	0.16	10.1	190	100	10	2	Yes	0.7	94.0
Rail Road-2	7.6	0.16	10.1	190	100	16	2	Marginal	1.0	49.5
River Site	7.6	0.16	4.6	86	47	6	0	Yes	0.6	96.6
Road Site	7.6	0.18	6.1	115	79	12	0	No	1.0	52.8
Showa Br2	7.6	0.16	4.3	80	39	4	10	Yes	0.5	99.9
Showa Br4	7.6	0.18	6.1	115	67	27	0	No	9.8	0.5
1968, M=7.5 earthquake-April 1										
Hososhima	7.5	0.242	2.895	53	45	8	36	No	1.1	21.2
1968, M=8.3 Tokachi-Oki earthquake-may 16										
Amoro Station	8.3	0.213	5.7	95	38	9	3	No	0.5	99.6
Hachinohe-2	8.3	0.23	6.1	115	76	28	5	No	6.8	4.3
Hachinohe-4	8.3	0.23	4	75	45	16	5	N0	1.0	46.9
Hachinohe-6	8.3	0.23	4	75	42	6	5	Yes	0.4	100.0
Nanaehamal	8.3	0.2	4	75	45	5	20	Yes	0.5	99.7
1971, M=6.6 San Fernando- Feb 9										
Junenile Hall	6.61	0.45	6.1	112	96	3.5	55	Yes	0.5	100.0
Van Norman	6.61	0.45	6.1	112	96	7.3	50	Yes	0.6	99.3
1975, M=7.0 Haicheng earthquake - Feb 4										
Panjin Chemical	7	0.2	8.2	155	89	9.1	67	Yes	0.9	55.4
Shuang Tai Zi	7	0.2	8.2	158	92	9	50	No	0.9	56.6
Ying Kou Glass	7	0.3	7.8	147	85	13	48	Yes	0.8	83.2
Ying Kou Paper	7	0.3	8.2	158	92	11	5	Yes	0.5	99.9
1976, M=7.5 Guatemala earthquake - Feb 4										
Amatitlan B-1	7.5	0.135	10.4	139	86	6	3	Yes	0.7	94.5
Amatitlan B-2	7.5	0.135	4.6	55	34	8	3	Marginal	0.9	51.5
Amatitlan B-3&4	7.5	0.135	10.7	137	71	16	3	No	1.1	29.2
1976, M=7.6 Tangshan earthquake -July 27										
Coastal Region	7.6	0.13	4.5	87	54	9	12	Yes	1.2	12.7
Le Ting L8-14	7.6	0.2	4.4	81	53	9.7	12	Yes	0.8	78.3
Luan Nan-L1	7.6	0.22	3.5	62	38	19.3	5	No	1.6	13.4
Luan Nan-L2	7.6	0.22	3.5	56	32	5.9	3	Yes	0.5	100.0
QING Jia Ying	7.6	0.35	5.3	102	59	17	20	Yes	0.9	62.5
Tangshan City	7.6	0.5	5.3	98	75	30	10	No	2.4	21.8
Yao Yuan Village	7.6	0.2	6.1	118	67	9	20	Yes	0.8	85.4
1977, M=7.4 Argentina earthquake - Nov 23										
San Juan B-1	7.5	0.2	8.2	142	106	9	20	Yes	0.8	89.0
San Juan B-3	7.5	0.2	11.1	199	156	13	5	Yes	0.6	98.3
San Juan B-4	7.5	0.2	3.7	63	39	14	4	No	0.8	75.6
San Juan B-5	7.5	0.2	3.1	53	44	14	3	No	1.0	34.1
San Juan B-6	7.5	0.2	5.2	90	56	6	50	Yes	0.7	94.3
1978, M=6.5 Miyagiken-Oki earthquake- Feb 20										
Arahama	6.5	0.1	6.4	121	67	10	0	No	1.8	0.1

Appendix A. Continue.

Site	M_w	a_{max}/g	Z	σ_{v0}^*	$\sigma'_{v0}{}^*$	N_{SPT}	FC	Liq	FS	PL
Hiyori-18	6.5	0.14	5.2	98	71	9	20	No	2.0	0.0
Ishinomaki	6.5	0.12	3.5	66	45	3.7	10	No	1.2	4.0
Kitawabuchi	6.5	0.14	3.4	62	59	11	5	No	2.1	0.0
Nakajima	6.5	0.14	6.1	115	79	12	3	No	1.7	0.5
Nakamura Dyke N4	6.5	0.12	2.8	53	30	4.7	5	Yes	1.3	23.7
Nakamura Dyke N5	6.5	0.12	3.4	63	42	7	4	No	1.4	1.0
Oiiri-1	6.5	0.14	6.4	106	85	9	5	No	1.5	0.9
Shiomi-6	6.5	0.14	4	75	60	6	10	No	1.4	0.8
Yuriage Br-1	6.5	0.12	4.3	80	56	4	10	No	1.3	2.4
Yuriage Br-2	6.5	0.12	2.5	46	34	10.1	7	No	2.4	0.0
Yuriage Br-3	6.5	0.12	4.3	80	42	8	12	No	1.5	0.8
Yuriagekami-1	6.5	0.12	5.5	99	63	2	60	No	1.3	0.8
Yuriagekami-2	6.5	0.12	4.3	80	47	11	0	No	1.8	0.2
1978, M=7.7 Miyagiken-Oki earthquake- June 12										
Arahama A-9	7.7	0.2	6.4	121	67	10	0	Yes	0.6	96.2
Hiyori-18, Site C	7.7	0.24	5.2	98	71	9	20	Yes	0.8	79.3
Ishinomaki-2	7.7	0.2	3.5	66	45	3.7	10	Yes	0.5	99.9
Ishinomaki-4	7.7	0.2	4.5	87	57	14.2	10	No	1.3	14.5
Kitawabuchi-2	7.7	0.28	3.4	62	59	11	5	Yes	0.7	86.2
Kitawabuchi-3	7.7	0.28	4.8	90	73	13.2	0	No	0.9	66.9
Nakajima2	7.7	0.24	4.6	86	65	10	26	No	1.0	48.2
Nakajima-18	7.7	0.24	6.1	115	79	12	3	Yes	0.7	91.8
Nakamura Dyke N1	7.7	0.32	3.4	63	39	19	4	No	1.4	31.7
Nakamura Dyke N4	7.7	0.32	2.8	53	30	4.7	5	Yes	0.3	100.0
Nakamura Dyke N5	7.7	0.32	3.4	63	42	7	4	Yes	0.4	100.0
Oiiri-1	7.7	0.24	6.4	106	85	9	5	Yes	0.6	98.4
Shiomi-6	7.7	0.24	4	75	60	6	10	Yes	0.6	99.3
Yuriage Br-1	7.7	0.24	4.3	80	56	4	10	Yes	0.4	100.0
Yuriage Br-2	7.7	0.24	2.5	46	34	10.1	7	Yes	0.8	68.6
Yuriage Br-3	7.7	0.24	4.3	80	42	8	12	Yes	0.5	99.6
Yuriage Br-5	7.7	0.24	7.3	138	78	17	17	No	1.1	40.1
Yurigekami-1	7.7	0.24	5.5	99	63	2	60	Yes	0.5	100.0
Yurigekami-2	7.7	0.24	4.3	80	47	11	0	Yes	0.6	94.7
Yurigekami-3	7.7	0.24	5.5	103	70	20	0	Yes	1.4	18.8
1979, M=6.5 Imperial Valley earthquake- Oct 15										
Heber Road A1	6.53	0.78	2.9	53	42	30.4	12	No	4.0	2.9
Heber Road A2	6.53	0.78	3.7	68	50	2	18	Yes	0.2	100.0
Heber Road A3	6.53	0.78	4	79	56	13	25	No	0.5	98.1
Kornbloom B	6.53	0.13	4.3	77	62	5	92	No	1.9	0.0
McKim Ranch A	6.53	0.51	2.1	38	32	3	31	Yes	0.4	100.0
Radio Tower B1	6.53	0.2	3.4	62	50	2	64	Yes	1.0	33.7
Radio Tower B2	6.53	0.2	2.3	40	38	11	30	No	2.4	0.0
River Park A	6.53	0.24	1.8	35	20	3	80	Yes	0.6	99.0
Wildlife B	6.53	0.17	4.6	87	54	7.1	30	No	1.4	1.6
1980, M=6.0 Mid-Chiba earthquake- Sep 24										
Owi-1	6	0.095	6.1	108	57	5	13	No	1.8	0.0
Owi-2	6	0.095	14.3	254	123	4	27	No	1.8	0.0
1981, M=5.9 West Morland earthquake- April 26										
Kombloom B	5.9	0.32	4.3	77	62	5	92	Yes	0.9	58.2
McKim Ranch A	5.9	0.09	2.1	38	32	3	31	No	2.9	0.0
Radio Tower B1	5.9	0.2	3.4	62	50	2	64	Yes	1.2	4.7
Radio Tower B2	5.9	0.2	2.3	40	38	11	30	No	2.8	0.0
River Park A	5.9	0.21	1.8	35	20	3	80	No	0.9	67.3
River Park C	5.9	0.21	4.3	83	45	11	18	No	1.5	2.4
Wildlife B	5.9	0.26	4.6	87	54	7.1	30	Yes	1.1	20.4
1982, M=6.9 Urakawa-Oki earthquake- Mar 21										
Tokachi	6.9	0.168	2.4	42	35	10	5	No	1.8	0.5

Appendix A. Continue.

Site	M_w	a_{max}/g	Z	$\sigma_{v0}{}^*$	$\sigma'_{v0}{}^*$	N_{SPT}	FC	Liq	FS	PL
1983, M=6.8 Nihonkai-Chubu earthquake- June 21										
Arayamotomachi	6.8	0.15	4.3	69	37	2.6	5	No	0.7	98.0
Arayamotomachi sand	6.8	0.15	9.2	158	77	13.1	0	No	1.4	8.1
Takeda Elementary	6.8	0.111	4.3	81	42	7.4	0	Yes	1.4	2.6
1983, M=7.7 Nihonkai-Chubu earthquake- May 26										
Aomori Station	7.7	0.116	5.7	95	38	9	3	Yes	1.0	41.7
Arayamotomachi	7.7	0.2	4.3	69	37	2.6	5	Yes	0.4	100.0
Gaiko Wharf B-2	7.7	0.227	7.5	123	53	7.7	1	Yes	0.4	100.0
Noshiro Section N-7	7.7	0.25	3.5	55	38	9.8	1	Yes	0.8	80.8
Takeda Elementary	7.7	0.283	4.3	81	42	7.4	0	Yes	0.4	100.0
Akita Station 1	7.7	0.205	2.8	52	41	12	3	No	1.2	15.5
Akita Station 2	7.7	0.205	2.8	53	41	8.5	3	No	0.9	58.6
Aomori Port	7.7	0.116	3.3	63	41	8	5	No	1.3	7.8
Gaiko 1	7.7	0.205	6.9	132	79	6.6	3	Yes	0.5	99.8
Gaiko 2	7.7	0.205	9.7	189	107	5.9	4	Yes	0.4	100.0
Hakodate	7.7	0.052	4.2	81	54	2.6	66	No	2.5	0.0
Nakajima No.1(5)	7.7	0.205	6.4	124	74	7.3	8	Yes	0.6	99.1
Nakajima No. 2(1)	7.7	0.205	7.1	136	81	10.4	3	Yes	0.7	92.9
Nakajima No. 2(2)	7.7	0.205	3.7	71	48	6	7	Yes	0.6	98.5
Nakajima No. 3(3)	7.7	0.205	6.0	115	71	7.3	2	Yes	0.6	98.8
Nakajima No. 3(4)	7.7	0.205	5.7	109	68	8	2	Yes	0.6	96.5
Ohama No. 1(1)	7.7	0.205	3.9	74	47	13	3	No	1.0	38.6
Ohama No.1(2)	7.7	0.205	3.4	64	42	15.9	2	No	1.6	9.9
Ohama No.1(3)	7.7	0.205	2.5	48	34	14.1	1	No	1.6	7.8
Ohama No.1(4)	7.7	0.205	5.1	99	60	25	3	No	8.6	1.2
Ohama No.1(5)	7.7	0.205	2.2	41	31	24.7	1	No	10.5	0.1
Ohama No.1(58-22)	7.7	0.205	4.4	85	53	13.2	2	No	1.1	31.4
Ohama No.2(2)	7.7	0.205	5.2	100	56	3.3	2	Yes	0.4	100.0
Ohama No.3(1)	7.7	0.205	5.4	103	63	4.8	2	Yes	0.5	100.0
Ohama No. 3(3)	7.7	0.205	5.4	104	64	3.7	2	Yes	0.4	100.0
Ohama No. 3(4)	7.7	0.205	3.9	73	49	5.2	2	Yes	0.6	99.6
Ohama No. Rvt (1)	7.7	0.205	4.5	86	55	15.8	2	No	1.5	12.8
Ohama No. Rvt (2)	7.7	0.205	6.6	127	77	17.3	4	No	1.2	27.5
Ohama No. Rvt(3)	7.7	0.205	3.5	67	46	18.3	0	No	2.1	4.2
1984, M=6.9 earthquake -Aug 7										
Hososhima	6.9	0.268	2.8	53	45	8	36	No	1.2	12.6
1987, M=6.2 Supersitition Hills earthquakes-1&2-Nov 24										
Radio Tower B1	6.22	0.09	3.4	62	50	2	64	No	2.4	0.0
Wildlife B	6.22	0.133	4.6	87	54	7.1	30	No	1.9	0.0
Heber Road A1	6.54	0.156	2.9	53	42	30.4	12	No	20.0	0.0
Heber Road A2	6.54	0.15	3.7	68	50	2	18	No	1.1	13.5
Heber Road A3	6.54	0.13	4	79	56	13	25	No	2.9	0.0
Kombloom B	6.54	0.174	4.3	77	62	5	92	No	1.4	0.5
McKim Ranch A	6.54	0.16	2.1	38	32	3	31	No	1.4	0.5
Radio Tower B1	6.54	0.2	3.4	62	50	2	64	No	1.0	34.3
Radio Tower B2	6.54	0.18	2.3	40	38	11	30	No	2.7	0.0
River Park A	6.54	0.19	1.8	35	20	3	80	No	0.8	81.0
River Park C	6.54	0.19	4.3	83	45	11	18	No	1.3	5.9
Wildlife B	6.54	0.206	4.6	87	54	7.1	30	Yes	1.1	14.3
1989, M=6.9 Loma Prieta earthquake										
Alameda Bay Farm	6.93	0.24	6.5	125	91	37	7	No	10.6	0.1
Faris Farm	6.93	0.37	6	106	92	9	8	Yes	0.6	99.4
General Fish	6.93	0.28	2.5	45	35	16.9	5	No	1.4	13.1
Hall Avenue	6.93	0.14	4.6	75	64	4.6	30	No	1.6	0.1
Marine Laboratory B1	6.93	0.28	4.6	87	65	11	3	Yes	0.8	83.3
Marine Laboratory B2	6.93	0.28	3.5	65	55	13	3	Yes	1.0	47.6
Marine Laboratory UCB	6.93	0.28	5.3	102	64	12	3	Yes	0.9	67.1

Appendix A. Continue.

Site	M_w	a_{max}/g	Z	σ_{v0}^{*}	$\sigma'_{v0}{}^{*}$	N_{SPT}	FC	Liq	FS	PL
MBARI No.3:EB-1	6.93	0.28	2	35	35	18	1	No	2.0	1.5
MBARI No.3:EB-5	6.93	0.28	3.4	63	47	12	1	No	1.2	19.0
MBARI No.4	6.93	0.28	3.4	62	48	18	5	No	1.3	14.4
MBARI Technology	6.93	0.28	3.4	62	48	12	4	No	0.9	65.9
Miller Farm CMF3	6.93	0.39	6.2	114	101	9.2	32	Yes	0.7	94.2
Miller Farm CMF5	6.93	0.39	7	130	108	20	13	Yes	1.0	44.6
Miller Farm CMF8	6.93	0.39	6	111	95	8.8	25	Yes	0.7	96.5
Miller Farm CMF10	6.93	0.39	8.4	158	105	19	20	No	0.9	56.2
POO7-2	6.93	0.28	6.3	121	89	14.4	3	Yes	0.8	77.5
POO7-3	6.93	0.28	6.3	121	89	16	3	Marginal	0.9	66.7
POR-2,3,4	6.93	0.18	5.9	97	73	4.3	50	Yes	1.1	24.1
Sandholt UC-B10	6.93	0.28	3	55	43	9.5	2	Yes	0.9	58.1
Sandholt2 UC-B10	6.93	0.28	6.1	115	73	26	5	No	6.8	2.0
SFOBB-1&2	6.93	0.27	6.3	118	86	7.5	8	Yes	0.6	99.0
State Beach UC-B1	6.93	0.28	3.4	61	46	6.3	1	Yes	0.6	97.3
State Beach UC-B2	6.93	0.28	4.9	90	67	12.8	1	Yes	1.0	37.5
Treasure Island	6.93	0.16	6.5	115	67	4.3	20	Yes	1.0	46.3
Wood Marine UC-B4	6.93	0.28	1.8	32	25	6.7	35	Yes	0.8	64.8
1990, M=7.7 Luzon earthquake- July 16										
Cerenan St.B-12	7.7	0.25	5	84	68	34.7	19	No	2.7	1.8
Perez Blv. B11	7.7	0.25	7.2	139	90	19.9	19	Yes	0.7	87.9
1993, M=7.6 Kushiro-Oki earthquake- Jan 15										
Kushiro Port Quay Site A	7.6	0.4	5.2	100	68	11.7	2	Yes	0.5	99.3
Kushiro Port Quay Site D	7.6	0.4	10.8	208	118	26.8	0	Yes	1.1	48.5
Kushiro Port Seismo St	7.6	0.47	3.8	65	47	17.4	5	Yes	1.0	50.9
1994, M=6.7, Northridge earthquake -Jan 17										
Balboa Blv. Unit C	6.69	0.84	8.5	156	143	13.6	50	Yes	0.4	99.9
Malden Street Unit D	6.69	0.51	9.3	154	101	24.1	25	No	1.9	20.8
Potrero Canyon C1	6.69	0.43	7.1	139	88	7.4	64	Yes	0.5	100.0
Wynne Ave. Unit C1	6.69	0.51	6.7	129	105	11	33	Yes	0.6	98.9
1995, M=6.9 Hyogoken-Nambu (Kobe) earthquake-Jan 16										
1	6.9	0.4	5.8	113	80	42.1	3	No	6.4	0.4
2	6.9	0.4	8	152	103	34.2	15	No	5.9	2.0
3	6.9	0.4	5.8	109	77	40	3	No	6.5	0.5
4	6.9	0.4	4.3	76	54	25.8	1	No	6.5	2.4
5	6.9	0.35	8.9	173	116	5.4	1	Yes	0.3	100.0
6	6.9	0.4	5.9	107	72	13.4	21	No	0.9	62.3
7	6.9	0.4	3.3	62	60	8	0	Yes	0.6	98.4
8	6.9	0.5	5	85	65	17.4	0	Yes	0.9	55.7
9	6.9	0.5	4.3	79	64	8.3	2	Yes	0.4	100.0
10	6.9	0.6	7.5	137	107	24.1	9	No	0.9	56.2
11	6.9	0.5	6.8	114	62	5.6	5	Yes	0.3	100.0
12	6.9	0.5	5.3	92	72	18.6	14	No	1.3	31.9
13	6.9	0.5	6.5	116	74	9.5	15	Yes	0.5	99.9
14	6.9	0.5	4.8	86	69	15	19	No	1.0	49.7
15	6.9	0.5	5.7	102	82	15.1	5	Yes	0.7	90.2
16	6.9	0.6	4.5	80	60	17.5	5	No	0.8	63.6
17	6.9	0.5	4.5	80	43	12.6	5	Yes	0.5	92.9
18	6.9	0.7	10.5	199	171	40.5	0	No	3.9	16.8
19	6.9	0.6	7.5	137	124	20	10	No	0.7	86.2
20	6.9	0.55	6	114	75	50.8	0	No	4.4	0.2
21	6.9	0.6	3.5	62	44	24.4	0	No	4.2	12.6
22	6.9	0.6	6	114	79	30.8	6	No	4.2	10.1
23	6.9	0.6	5	92	72	18.1	10	No	0.9	64.2
24	6.9	0.5	3.5	63	51	18	0	Yes	1.1	43.7
25	6.9	0.7	3.5	64	50	27.5	3	No	4.0	10.0
26	6.9	0.6	3.5	63	37	26	0	No	3.5	7.1

Appendix A. Continue.

Site	M_w	a_{max}/g	Z	σ_{v0}^{*}	$\sigma'_{v0}{}^{*}$	N_{SPT}	FC	Liq	FS	PL
27	6.9	0.6	2.5	43	29	27.6	10	No	4.0	1.6
28	6.9	0.4	3.5	62	44	14.3	8	Yes	0.9	58.9
29	6.9	0.4	3.8	67	49	12.4	0	Yes	0.7	85.9
30	6.9	0.6	8.5	146	78	30.5	10	No	3.4	8.5
31	6.9	0.6	4	73	46	34.8	0	No	3.8	0.8
32	6.9	0.5	3.5	61	41	20.1	6	No	2.0	23.7
33	6.9	0.5	8	142	83	21.3	50	No	2.3	19.1
34	6.9	0.4	7	124	73	18.3	9	Yes	1.0	54.1
35	6.9	0.5	4.5	79	55	12.3	6	Yes	0.6	95.0
36	6.9	0.6	3.5	61	36	21.2	3	No	3.2	22.7
37	6.9	0.35	5	89	79	15	0	Yes	1.0	41.0
38	6.9	0.5	8	143	94	15.1	5	Yes	0.5	96.9
39	6.9	0.6	4.5	84	66	47	0	No	4.8	0.2
40	6.9	0.6	3.5	66	59	32.5	0	No	5.3	2.8
41	6.9	0.4	4.1	71	50	9.2	10	Yes	0.6	96.7
42	6.9	0.4	5	84	46	7	10	Yes	0.4	100.0
43	6.9	0.35	4.7	80	55	10	20	Yes	0.9	66.0
44	6.9	0.4	4	67	43	4.4	5	Yes	0.3	100.0
Ashiyama A	6.9	0.4	5.2	97	80	16.6	18	No	1.3	24.0
Ashiyama C-D-E	6.9	0.4	8.8	166	115	10.9	2	Yes	0.5	99.9
Port Island Borehole Array	6.9	0.34	7.8	149	96	5.7	20	Yes	0.5	100.0
Port Island Tanahashi	6.9	0.4	8.5	159	125	20.2	20	No	1.2	29.7
Port Island Improved site	6.9	0.4	10	189	140	18.2	20	No	0.9	61.1
Port Island Watanable	6.9	0.4	9.5	179	135	30.9	20	No	6.2	3.8
Port Island Site I	6.9	0.34	10	192	123	9.7	20	Yes	0.6	98.9
Rokko Island Building D	6.9	0.4	7.5	141	107	14.8	25	Yes	0.8	68.5
Rokko Island Site G	6.9	0.34	11.5	219	146	12	20	Yes	0.6	95.9
Torishima Dike	6.9	0.25	4.7	93	46	8.5	20	Yes	0.8	75.0