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Technical Note

Nonlinear Soil-Structure Interaction Effects on Building Frames: A Discussion on the Seismic Codes

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ABSTRACT

Recent seismic codes include design requirements in order to take soil-structure interaction (SSI) into account for realistic modelling of structures. The paper investigates the performance of multi-story building-foundation systems through a Winkler-based approach. A set of 4-, 8-, 12- and 16-story steel moment resisting frame buildings on three soil types with shear wave velocities less than 600m/s subjected to actual earthquake records with a probability of exceedance of 10% in 50 years are modeled with and without SSI. It is observed that the performance level of frames supported by flexible foundation, particularly at soft soil sites, may alter significantly in comparison to fixed-base structures. Moreover, the nonlinear foundation is found to have a significant effect on the force and displacement demands. A comparison and brief discussion on the design guidelines for consideration of flexible foundation behavior is also included.

Keywords:
Soil-structure interaction; Seismic code; Building frame; Nonlinear analysis

1. Introduction

When the ground is stiff enough, the dynamic response of the structure will not be influenced significantly by the supporting soil properties during the earthquake, and the structure can be analyzed under the fixed-base condition. However, for the structure resting on a flexible medium, the dynamic response of the structure will be different from the fixed-base condition owing to the interaction between the soil and the structure [1]. In general, soil-structure interaction (SSI) effects can be summarized as follows: reduction of the natural frequency of the system, increase in damping, increase of the lateral displacement, and change in the force demands of the structure [2-3]. For stiff structural systems such as shear wall and braced frame founded on soil, ignoring the influence of foundation movements could lead to a significant misestimation of the fundamental frequency in the system [4-6]. Analyses conducted for various soil and structure conditions showed that this influence depends mainly on the soil-structure relative rigidity. The increased period and damping of the soil-structure system largely leads to the consequences that the SSI has beneficial effects and is mostly ignored in seismic design of buildings. This conclusion could be misleading since ignoring base flexibility may over or under predict seismic response of the structure, depending primarily on the characteristics of the ground motions. It was shown that the SSI can play a detrimental role and neglecting its influence could lead to unsafe design [7-8].

Although not widely used in practice, engineering guidelines exist for simple evaluation of SSI effects.

Recent code-compliant seismic designs for SSI systems, such as NEHRP [9] and ASCE 7 [10], are based on the approximation in which the predominant period and associated damping of the corresponding fixed-base system are modified. On the other hand, ATC 40 [11] and FEMA 356 [12] partially address the flexible foundation effect through including the stiffness and strength of the soil components of the foundation (Winkler-based model) in the structural analysis. However, none of these procedures address the shaking demand on the structure relative to the free-field motion caused by kinematic interaction or the foundation damping effect. Guidance on including kinematic interaction effects are given in FEMA 440 [13] and ASCE 41 [14].

Various methods have been used to model the behaviour of structures on shallow foundations [15-17]. However, the application of simple methods such as the Winkler approach is preferred in practical SSI problems. In this context, Beam-on-Nonlinear-Winkler-Foundation (BNWF) method proposed by Harden et al. [18], Harden and Hutchinson [19], and later by Gajan et al. [20], has been widely applied in recent studies due to its relative simplicity and minimal computational effort [e.g., 21-23]. In the present article, an extensive parametric study is carried out to evaluate seismic response of low-to-mid-rise steel moment resisting frame (MRF) buildings in which the BNWF method is used to model the behavior of shallow foundation. The numerical results indicate that the SSI can alter the force and drift demands, which is significantly important to the performance-based design of structures. Furthermore, the predominant period and damping ratio defined in design procedures such as ASCE, NEHRP, etc. to include the SSI effect is evaluated, and also the accuracy in the determination of the design earthquake forces, if models correspond to flexible-based conditions, is studied.

2. Soil-Structure System

Steel MRFs with 4, 8, 12 and 16 stories located on hypothetically soft, medium and hard sites were considered, as shown in Figure (1). Buildings were designed as special frames based on the requirement of Iranian national building codes [24-25] and Iranian code of practice for seismic resistant design of buildings (Standard No. 2800) [26]. The dead and live loads of 600 and 200 kgf/m^2, respectively, were used for gravity loads. The importance factor

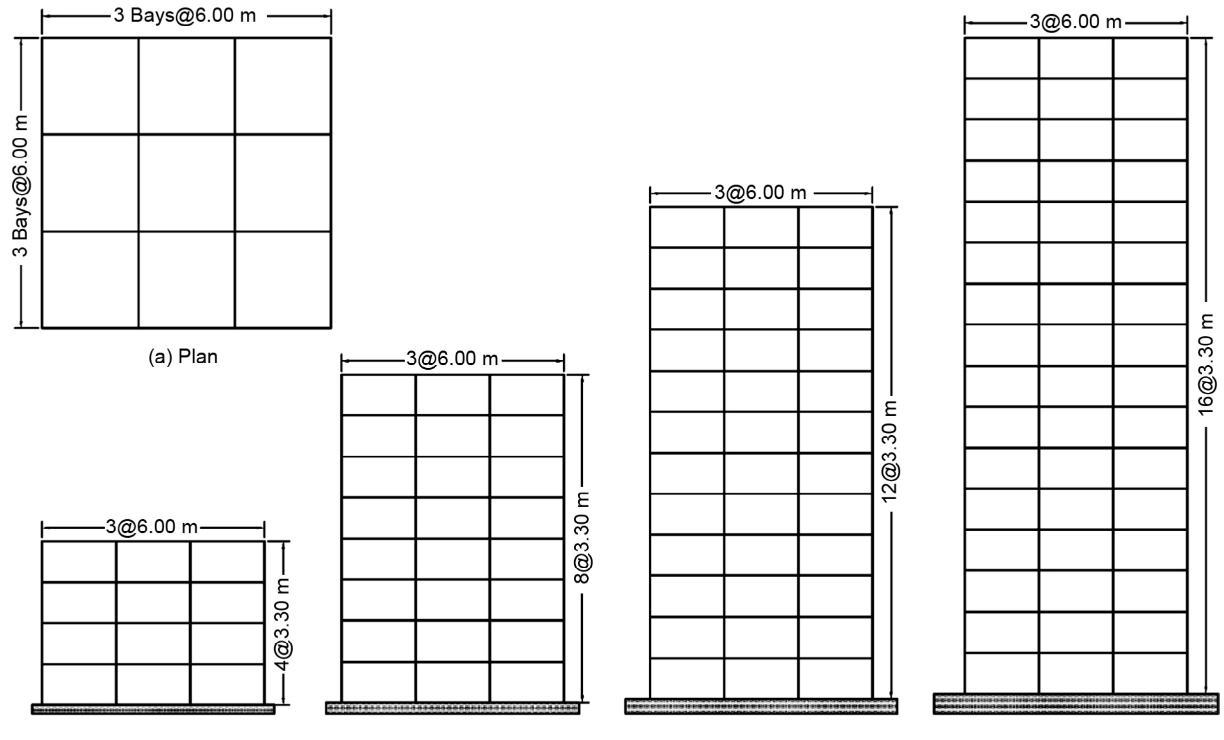


Figure 1. Configuration of building models.

of $I = 1$, response modification factor of $R = 7.5$ and seismic zone factor of $A = 0.30$ have been considered for the frames design. Columns are supported by strip footings resting on different soil conditions including types II ($375 < v_s < 750$m/s), III ($175 < v_s < 375$m/s) and IV ($v_s < 175$m/s) according to classification of the Standard No. 2800. Note that for soils with shear wave velocity greater than 600 m/s, the effect of SSI is not considerable [16, 27]. The details of various soil parameters are as tabulated in Table (1), which is extracted from the actual geotechnical projects (personal communication). Table (2) summarizes the characteristics of the footings in this study, where, q_{all} is allowable bearing capacity of soil, Ks is the subgrade soil reaction. B and H are the width and thickness of foundation, respectively.

3. Selection of Ground Motions

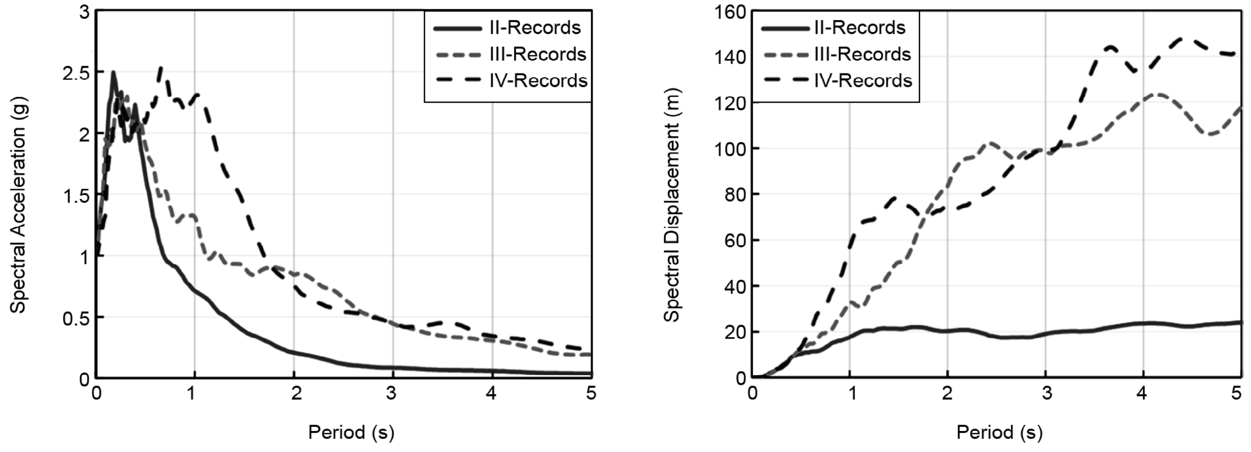
A database of 21 recorded ground motion time histories with a wide range of intensity, duration, frequency contents and earthquake magnitudes (i.e., Mw 5.9-7.6) has been compiled from well-known studied seismic events. All motions, recorded at closest fault distances greater than 10 km, are divided into three groups for stiff, medium and soft soil sites. Each soil type is represented by an ensemble of seven ground motions taken from the PEER NGA database [28]. Figure (2) show the 5% damped mean acceleration response spectra and displacement response spectra for the three soil types, respectively. More details of these

Table 1. Details of soil parameters.

Soil Type	Ø (Degree)	C (kgf/cm^2)	υ	γ (kgf/m^3)	G (kgf/cm^2)	v_S (m/sec)
II	30	0.15	0.35	2100	6707	560
III	27	0.10	0.40	1900	732.2	275
IV	15	0.03	0.40	1700	339.3	150

Table 2. Foundation characteristics used in analyses.

Building Model	Footing Type	B (m)	H (m)	E_c (kg/cm^2)	f_c (kg/cm^2)	υ	f_Y (kg/cm^2)	q_{all} (kg/cm^2)	K_s (kg/cm^3)
4-Story	Strip	1.0~1.5	0.65	218819	210	0.2	4000	1.0~2.0	1.5~2.4
8-Story	Strip	1.7~2.8	0.90	218819	210	0.2	4000	1.0~2.0	1.5~2.4
12-Story	Strip	2.6~4.0	1.20	218819	210	0.2	4000	1.0~2.0	1.5~2.4
16-Story	Strip	3.5~5.0	1.50	218819	210	0.2	4000	1.0~2.0	1.5~2.4



(a)

(b)

Figure 2. (a) Mean acceleration response spectra, and (b) mean displacement response spectra; plotted as a function of period for 5% damping at soil types II, III and IV.

motions can be found in [29]. The earthquake records have been separately scaled according to the procedures described in Iranian Standard No. 2800 at seismic hazard levels of probability of exceedance 10% in 50 years.

4. Numerical Modeling

The computational model of the soil-foundation-structure systems were developed using the OpenSees finite element software [30]. The structural members, beams and columns, are modelled as nonlinear beam-column elements with a kinematic material hardening of 3%. The development of realistic numerical model of the foundation with its supporting subgrade soil has been recognized as an important and complex problem in earthquake engineering. Allotey and Naggar [31] elaborated a Winkler-based approach utilizing multi-linear, no-tension backbone curves. Besides, Harden and Hutchinson [19] developed a Winkler-based method using pile-calibrated nonlinear backbone curves of Boulanger et al. [32] to model shallow strip foundations, which can capture its rocking behaviour. The corresponding Beam-on-Nonlinear-Winkler-Foundation model was updated based on shallow footing load tests by Raychowdhury and Hutchinson [33] and implemented into the framework of OpenSees.

In this paper, the nonlinear response of MRF buildings is studied with two comparative base conditions. First, rigid base condition that means the foundation system is assumed to be fixed against all the movements (NSSI model). Second, SSI case in which foundation uplifting and soil plasticity are included and the soil-foundation interface is modelled as nonlinear Winkler springs. Vertical and sliding stiffness are selected based on recommendations by Gazetas [34], while vertical load bearing capacity is calculated after Terzaghi [35] when using foundation shape and depth factors proposed by Meyerhof [36]. Tension capacity of the soil is assumed as 10% of the compression capacity, q_{ult}, for soil modeling. The stiffness intensity and the spring spacing along the footing length are chosen based on Harden and Hutchinson [19].

5. Results and Discussions

In this section, an eigenvalue analysis followed by a dynamic time history analysis is performed to understand the behavior of the nonlinear structure incorporating the nonlinear SSI. They capture both geometrical (uplift, sliding and rocking motions) and material nonlinearities. Newmark linear acceleration method is used for conducting the transient analysis with solution parameters of $\gamma = 0.5$ and $\beta = 0.25$. A Rayleigh damping of 5% is assumed for the structure vibrating in its first and third modes. The following subsections discuss the effects of subsoil rigidity on inelastic seismic response of moment resisting frames and their performance levels. In order to reach a systematic criterion for analysis of SSI effects, the average responses were evaluated and presented. A comparison is also performed to evaluate the current regulations in estimating the seismic demand parameters for soil structure systems.

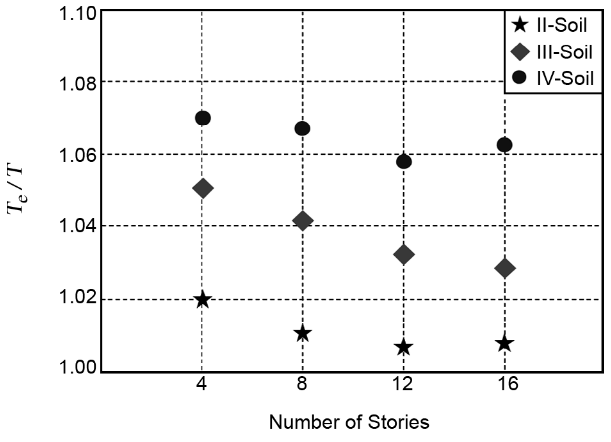
5.1. Period Lengthening

The vibration period is an important parameter of a structure to estimate its seismic demand. Modern building codes generally use the period ratio (flexible-base period, T_e to fixed-base period, T) of buildings to assess their response to seismic loadings. On the basis of ASCE 7 [10] and Standard No. 2800 [26], the effective period (T_e) shall be determined as follows:

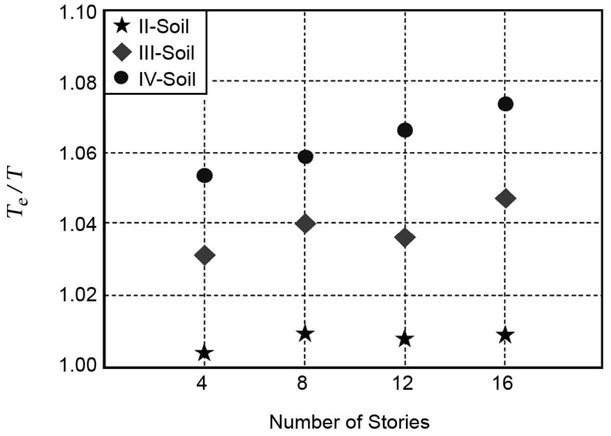
$$\frac{T_e}{T} = \sqrt{1 + \frac{K}{K_x} + \frac{K\bar{h}^2}{K_\theta}} \tag{1}$$

where K is the stiffness of the structure fixed at the base, $\bar{h}$ is the effective height of the structure, K_x and K_θ are the translational and the rocking stiffnesses of the foundation, respectively. Herein, the stiffness terms K, K_x and K_θ have been computed using guidelines given in FEMA 440 [13]; the soil properties are as presented in Table (1).

Figure (3) provides the fundamental vibration period of the building models with different base conditions using both simulation and Eq. (1). As shown in Figures (3a) and (3b), the effective period computed from SSI model is longer than those from rigid base model, due to the contribution of the rocking motion in the first mode. Results of the eigenvalue analysis indicate that fundamental period ratio is higher than 1.0, ranging from 1.02 to 1.07 for four-story model; from 1.01 to 1.07 for



(a)



(b)

Figure 3. Fundamental period ratios of the studied frames using (a) simulation, and (b) design code.

eight-story model; from 1.01 to 1.06 for 12-story model; and from 1.01 to 1.06 for 16-story model. It is observed that the period elongations of the MRFs obtained from simulation are in agreement with those obtained from the design code. Note that as the soil stiffness decreases, fundamental period response ratio increases. It is also noteworthy to mention that the buildings height has little effect on the period ratio, and can thus be neglected for evaluating the eigenvalue properties of the system. In general, the fundamental frequency of the soil-foundation-structure system is close to that of a fixed-base structure for the given MRF buildings. Nevertheless, the flexibility introduced by the soil-foundation system will play an important role in altering the overall force and displacement demand of the buildings, as will be demonstrated later in this paper.

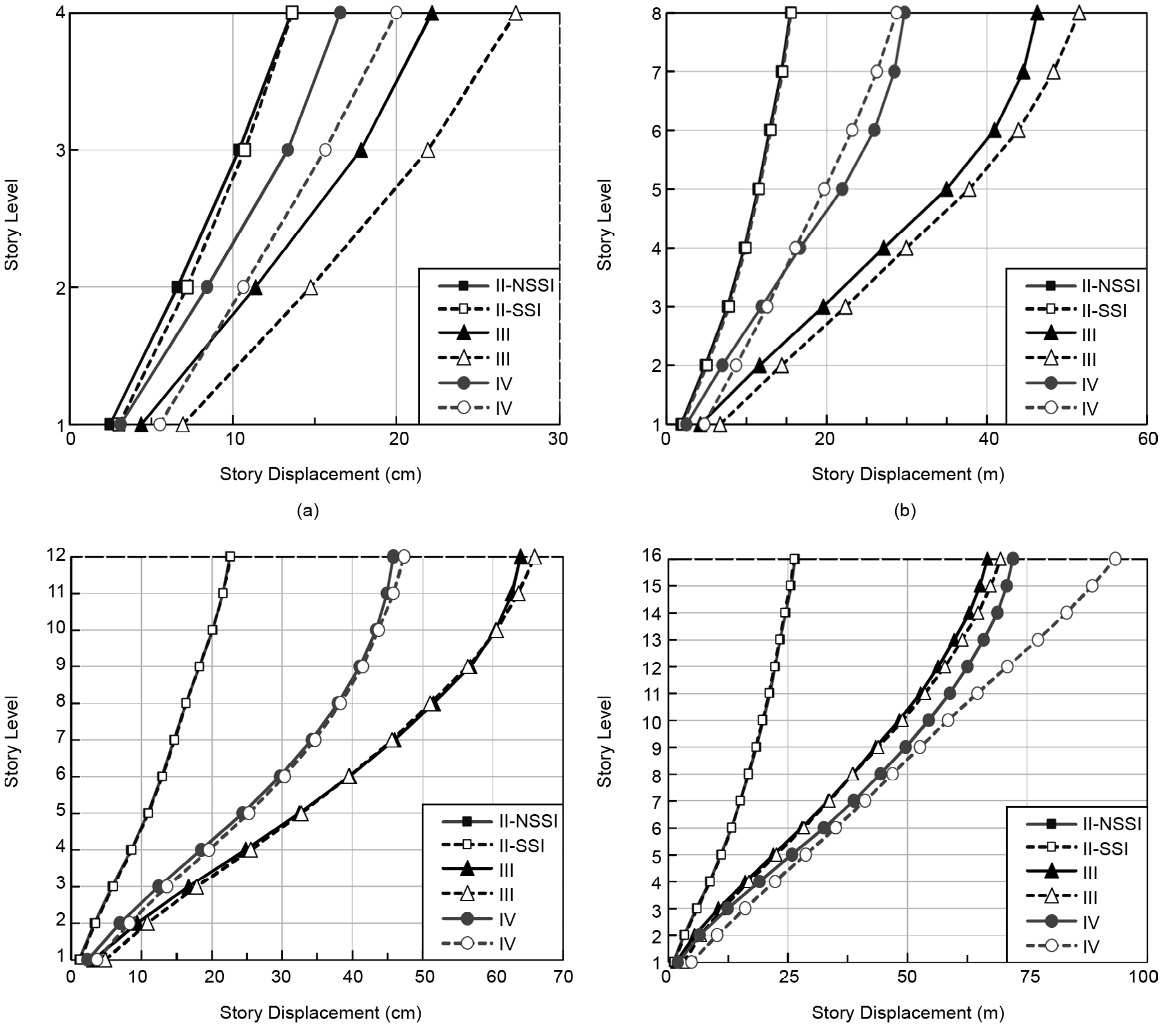
5.2. Story Deflection

The objective of inelastic seismic analysis method is to estimate directly and accurately the magnitude of inelastic structural deformations. The average peak absolute displacement at the floor level in the direction of applied acceleration building frame models are depicted over buildings height in Figure (4). As shown, the story displacement increases in SSI models, the increases are significant in foundations located on soft soil. Moreover, story displacement profile increases nonlinearly with the structural height. The increase in story deflection occurs due to the overall reduction in the global stiffness resulting from the induced foundation movements. This trend of increase in displacement demand may be expected, looking at the displacement response spectra in Figure (2b). It is necessary to note that the story deflection increases over all stories in the SSI model as seen in Figure (4). However, the rate of increase becomes higher for the 1st and 2nd stories. In other words, lower stories are more affected with SSI than the other stories.

5.3. Story Drift

Performance levels describe the state of structures after being subjected to a certain hazard and are classified as Immediate Occupancy (IO), Life Safety (LS) and Collapse Prevention (CP) for the selected earthquake hazard levels of probability of exceedance of 50%, 10% and 2% in 50 years, respectively [12]. The three above-mentioned qualitative performance levels are related to the corresponding quantitative maximum story drift ratio of 0.7%, 2.5% and 5%, respectively. Story drift ratios over the building's height are introduced in Figure (5) for different soil conditions using average envelope of time history analyses. According to Figures (5), seismic SSI tends to increase the story drifts in comparison with rigid base conditions. For example, the maximum average drift ratio of the fixed-base 16-story model for 10% in 50 years ground motions are measured to be 1.1%, 1.8% and 2.1% for the soil type II, III and IV respectively; while, the corresponding values for the flexible-base are 1.1%, 2.1% and 2.6%, respectively. Thus, the effects of SSI induce increases by up to 24% in the simulated story drifts. As a result, SSI may shift the performance level of a structure from life safe zone to near collapse level.

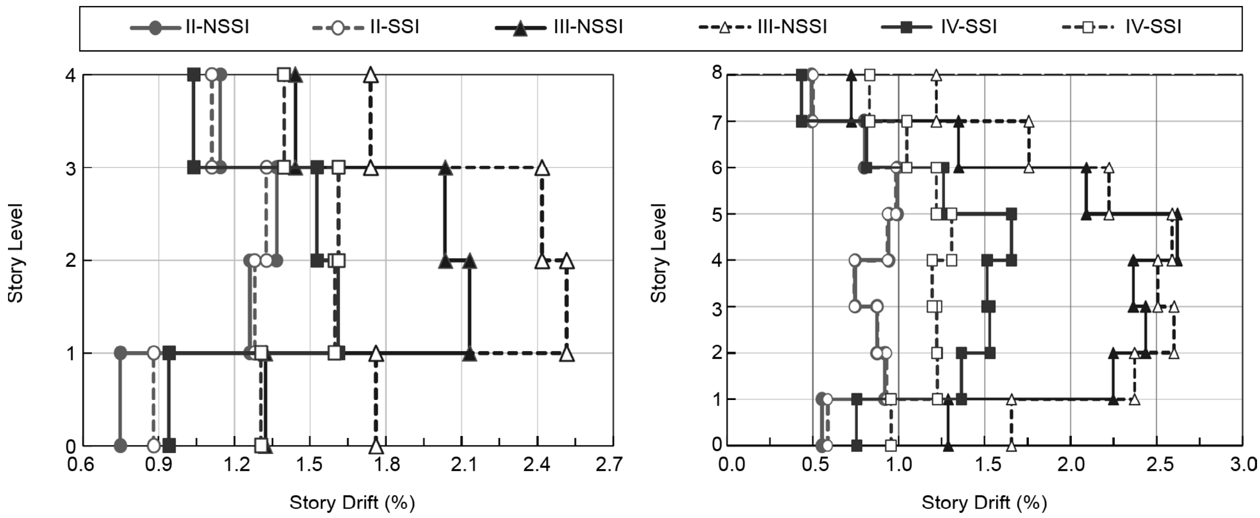
As shown in Figures (5), the drift demands



(a) (b)

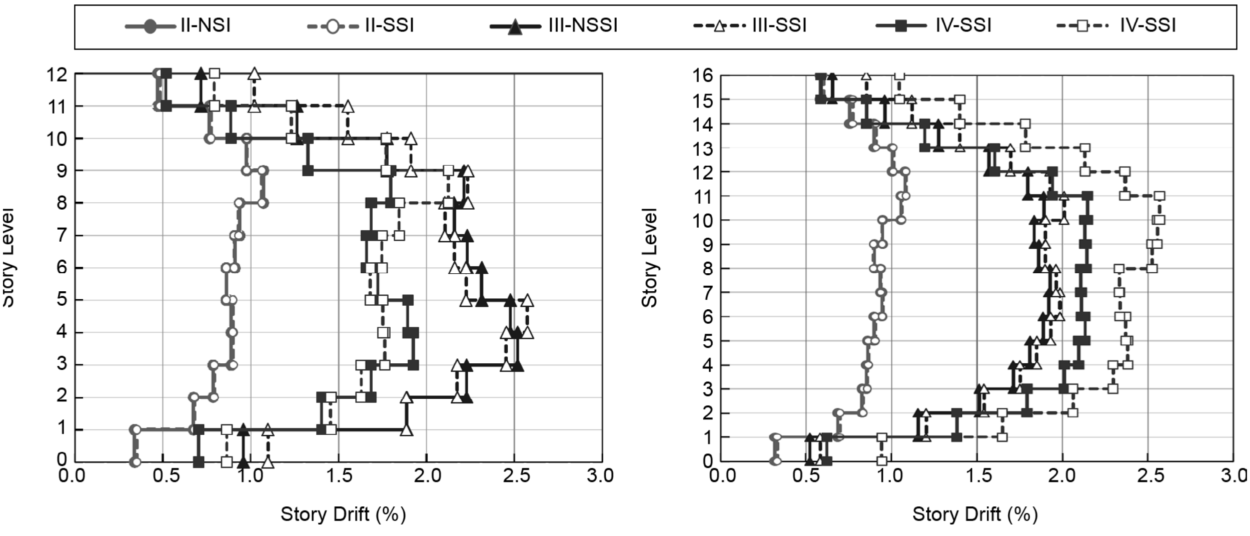
(c) (d)

Figure 4. Story deflection distributions for (a) 4-story, (b) 8-story, (c) 12-story, and (d) 16-story frames founded on site classes II, III and IV.



(a) (b)

Figure 5. Story drift ratio distributions for (a) 4-story and (b) 8-story16-story frames founded on site classes II, III and IV.



(c)

(d)

Figure 5. Story drift ratio distributions for (c) 12-story and (d) 16-story frames founded on site classes II, III and IV.

increase for flexible strip foundation as the underneath soil stiffness decreases. This increasing trend is more significant in the upper and lower stories. Besides, the SSI gets more significant effect on story drift ratio as the number of stories in the model increases. Since the drift demand is an important parameter for the design of structural members, it is very likely that the members are designed under-conservatively in the absence of incorporation of nonlinear SSI. Such a big difference in story displacements and drifts is not negligible; thus, the effect of SSI must be taken into account in dynamic analyses.

5.4. Shear Demand

Figure (6) presents the average structural inter-story shear demand in comparison with rigid base condition. The effect of foundation compliance is also evident here, indicating that softer foundation systems have the potential for greater reduction in structural shear forces due to the capacity mobilization of a larger number of mechanistic springs at the soil-foundation interface. In Figure (6), the shear demand ratio of low-rise MRFs (i.e. four and eight-story models) is almost less than one; while for the mid-rise models (i.e. 12 and 16-story models), the shear demands of flexible-bases are greater than rigid base conditions.

Current analysis procedures in the seismic regulations address the effective base shear (Ve) through Eq. (2) to account for the effects of SSI.

$$V_e = V - \Delta V \tag{2}$$

The reduction (ΔV) is computed as follows and shall not exceed 0.3 V and 0.15 V according to ASCE 7 and Standard No. 2800, respectively.

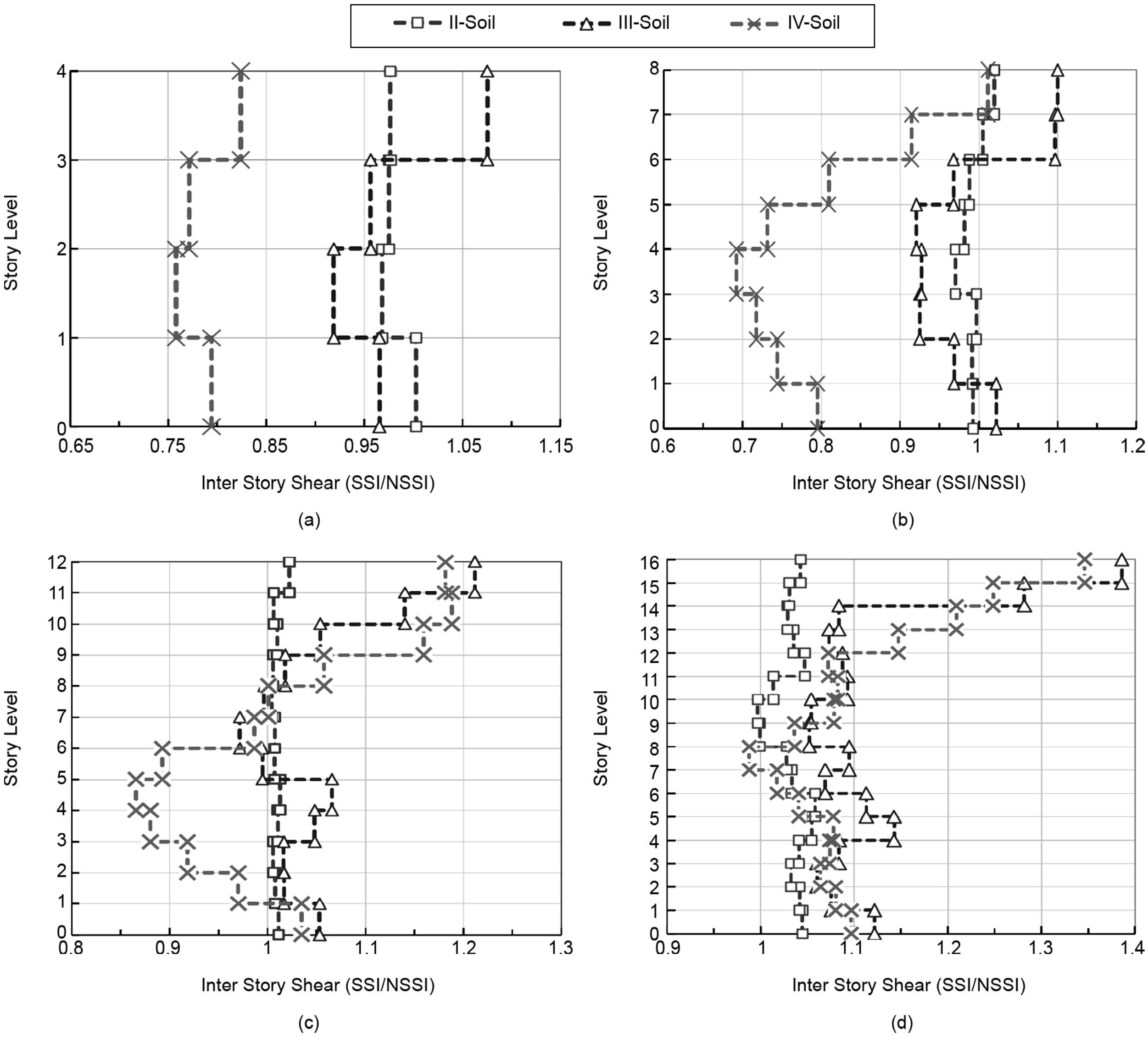
$$\Delta V = \left[C - \tilde{C}\left(\frac{0.05}{\tilde{\beta}}\right)^{0.4} \right]\overline{W} \tag{3}$$

where C is the seismic design coefficient computed using the fundamental natural period of the fixed-base structure (T), $\tilde{C}$ is the value of C computed using the fundamental natural period of the flexibly supported structure (Te) defined from Eq. (1), $\tilde{\beta}$ is the fraction of critical damping for the structure foundation system determined by Eq. (4), and $\overline{W}$ is the effective seismic weight of the structure.

$$\tilde{\beta} = \beta_0 + \frac{0.05}{\left(\frac{T_e}{T}\right)^3} \tag{4}$$

where β_0 is the foundation damping factor.

It was found that the effective base shear through the above code definition of SSI is slightly smaller than the base shear obtained from the fixed-base MRF buildings, in which the ratio of V_e / V is close to unity. On the other hand, Figure (7) shows the structural dynamic responses in terms of the peak base shear demand ratios for the soil type II, III and IV. It can be observed that the peak shear at the base of the columns reduces as much as 21% for the low-



(a) (b) (c) (d)

Figure 6. Story shear ratio distributions for (a) 4-story, (b) 8-story, (c) 12-story, and (d) 16-story frames founded on site classes II, III and IV.

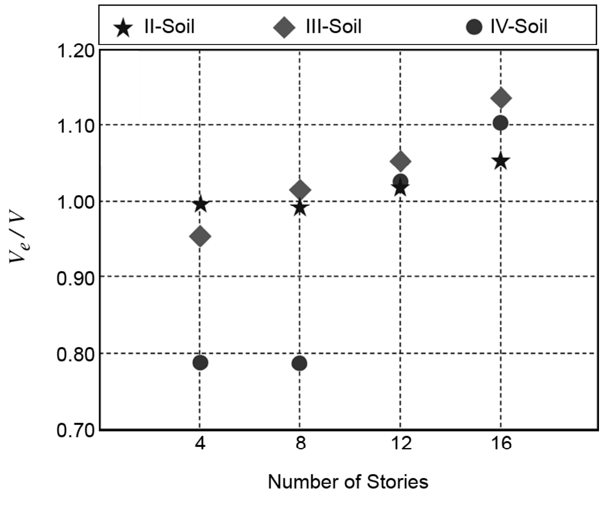


Figure 7. The fixed and flexible-base peak base shears of the studied frames on site classes II, III and IV.

rise frames, when nonlinearity at the soil-foundation interface is considered. While for 12 and 16-story models, the peak base shear increases as much as 13% when soil flexibility is considered. Depending on the first result, rigid base assumption would lead to an over-conservative estimation of the base shear especially as the soil gets softer. Depending on the second result, modeling the soil-foundation interface as fixed can exhibit an under-conservative estimation of the base shear. This implies that the change in base shear is related to the change in the characteristics of chosen structure and/or ground motion. The obtained results indicate that the earthquake demand evaluation can be based on a quite different scenario from those of indicated in the current seismic codes in which only base shear may reduce for the effect of SSI. Hence, additional work in this area of design guidelines is warranted.

6. Conclusions

In this study, the effects of SSI were analyzed for typical shallow foundations supporting multi-story buildings on various soil conditions, given the occurrence of 10% in 50 year ground shaking events. It was concluded that the dynamic SSI plays a considerable role in seismic behavior of low-to-mid-rise building frames including substantial increase in the lateral deflections and inter-story drifts and changing the performance level of the structures. If SSI is not taken into account in analysis and design properly; the accuracy in assessing the structural safety, facing earthquakes, could not be reliable. Thus, considering SSI effects in the seismic design of regular MRFs, particularly when resting on soft soil deposit, is essential. In addition, the code procedure to determine the effective earthquake demand parameters was evaluated for soil-structure systems. The investigation found that the ASCE and 2800 Standards provide conservative base shears at short MRF periods and unacceptable results for medium structural periods.

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